

St. Patrick's High School, Keady Mathematics Department

GCSE Mathematics Practice Booklet

M4

<u>Topic 2 – Algebra l</u>

The Language of Algebra

Expressions

Equations

Algebraic Fractions

Questions taken from CCEA Past Papers Mark Scheme included at the end of this booklet



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Factorise fully each of the following:	
(a) $12a + 6$	
	Answer[1]
(b) $y^2 - 6y$	
	Answer[1]
(c) $b + b^2$	
	Answer[1]

Expand and simplify

Q1

Q2

4(2x-3) - 2(x-5)

Answer [2]

Q3	Expand and simplify	2y(3y-7) - 8y
	1 1 2	

Answer [3]

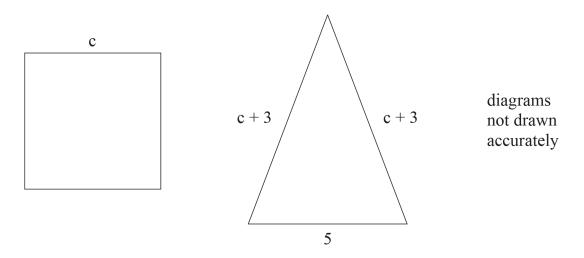
Q4 Solve 4 + 3(2x - 5) = x + 9

Answer *x* = _____ [3]

Q5 Solve 5(3x-2) = 7x+4

Answer *x* = _____[3]

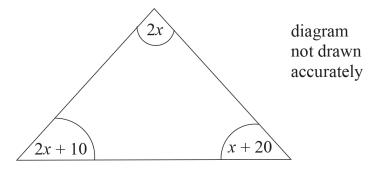
The diagrams below show a square and an isosceles triangle.



They have the same perimeter.

By forming and solving an equation, work out the perimeter.

Answer _____ [4]



Form and solve an equation to work out the size of the smallest angle in the triangle above.

Answer smallest angle = _____° [3]

(a) Simplify

Q8

5(t-2) - 3(4-2t)

Answer _____ [2]

(b) Expand and simplify (e + 4)(e - 7).

Answer _____ [2]

Q9 (a) Expand and simplify 3(4x+3)(2x-1)

(b) Factorise $t^2 - 49$

Answer _____ [1]

Answer _____ [3]

Answer [2]

Q11 (a) Expand $(3x - y)^2$

Answer [2]

(b) Factorise $x^2 - 25$

Answer [1]

Answer _____ [2]

Q13 (a) Factorise (i) 6a + 15Answer _____[1] (ii) $4x - x^2$ Answer _____[1] (b) Solve 6x - 7 = 14 - xAnswer x =____[3] (c) Simplify $\frac{y}{3} - \frac{y}{5}$

Answer [2]

Factorise each of the following.

(a) $10cp^2 - 4cp$ Answer _____ [2] (b) $y^2 - 1$ Answer _____ [1] (c) $k^2 - 2k - 3$

(d)
$$(x-2)^2 + 5(x-2)$$

Answer _____ [2]

Answer [2]

Q15 Factorise $y^2 - 6y + 8$

Answer _____ [2]

Q16	Expand and simplify	(n-6)(n+2)
	Expand and simplify	(p - 0)(p + 2)

		Answer	[2]
Q17	Factorise		
	(a) $p^2 - 3p$		
		Answer	[1]
	(b) $100 - t^2$		
		Answer	[1]
	(c) $x^2 + 2x - 15$		
		Answer	[2]
		Answer	[2]

Q18 (a) Expand and simplify (2x-5)(3x+2)

(b) Simplify
$$\frac{x^2 - 49}{2x - 14}$$

Answer _____ [2]

Q19

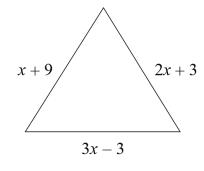
(a) Factorise $x^2 + 2x - 35$

Answer [2]

(b) Hence, or otherwise, solve the equation $x^2 + 2x = 35$

Answer *x* = _____ [1]





The diagram shows an equilateral triangle.

Form and solve an equation to calculate the **perimeter** of the triangle.

Equation _____[1]

Answer perimeter = [3]

Q21 Solve the equation $\frac{2x+3}{4} + \frac{x-1}{3} = 5$ Show all your work.

Answer *x* = _____ [4]

Q22 Solve
$$\frac{x+3}{2} = \frac{5x}{6}$$

Answer x = [4]

Q23 Solve

$$\frac{a-1}{4} + \frac{a+1}{8} = \frac{3}{2}$$

Give your answer as a mixed number.

Answer a = [4]

Solve

$$\frac{2x-1}{5} + \frac{4x+5}{3} = \frac{20}{3}$$

A solution by trial and improvement will not be accepted.

Answer _____ [4]

Q25 Mary and Anne both go to a shop, each with the same amount of money.

Mary buys 7 bars of chocolate at *y* pence each and receives 19p change.

Anne buys 5 bars of chocolate at *y* pence each and receives 65p change.

By forming and solving an equation, work out how much money each girl had going to the shop.

You must show all your working.

A solution by trial and improvement will not be accepted.

Equation		[1]
Answer: Each girl had	going to the shop.	[3]

Q26

Solve $\frac{1}{4}(y+3) - 2 = \frac{1}{2}(3-2y)$

Answer y = [4]

Q27 Simplify $\frac{5t}{6} - \frac{t}{2}$

Answer [3]

Q28 Solve the equation

$$\frac{2x-1}{3} + \frac{x+2}{2} + \frac{x}{6} = 8$$

Show all your working clearly.

A solution by trial and improvement will not be accepted.

Answer *x* = _____ [5]

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Solve the equation

$$\frac{2(3x+2)}{5} - \frac{(3x-4)}{3} = \frac{2}{15}$$

Show all your working out clearly.

A solution by trial and improvement will not be accepted.

Answer x = [4]

Q30 Simplify
$$\frac{x^2 - 2x}{3} \times \frac{6}{x^2 + 2x - 8}$$

Answer _____ [3]

Q31

Simplify $5 - \frac{x+10}{2}$

Answer _____ [2]

Simplify $\frac{x^3 - 3x^2}{x^2 - 9}$

Answer [3]

Q33

Factorise fully $100ax^2 + 65axy - 75ay^2$

Answer _____ [3]

Answer [2]

(b) Simplify $\frac{x^2 + 4x}{x^2 - 16}$

Answer [3]

(c) Factorise $8ax^2 - 26axy + 15ay^2$

Answer [3]

Answer _____ [2]

Q36 Simplify
$$\frac{6a^2 + 4a - 16}{8 - 2a^2}$$

Answer [3]

Answer [3]

(b) Using your answer from above and given that $a \neq 0$ solve

 $30ax^2 + 5axy - 60ay^2 = 0$

to find two answers for x in terms of y.

Answers x =_____, x =_____[2]

Q38

Simplify fully

$$\frac{3x^2 - 6xy}{4x^2 - 8xy - 3xy + 6y^2}$$

Answer _____ [2]

Q39

Factorise $15x^2 + 2xy - 8y^2$

Answer [2]

Q40 Factorise fully $8x^2 - 50y^2$

Answer _____ [3]

Q41 (a) Factorise $2a^2 + 7ab - 4b^2$

Answer _____ [2]

(b) Simplify the following

$$\left(\frac{x+1}{2x-1} + \frac{3x-4}{x-4}\right) \times \frac{2x-1}{x}$$

Answer [4]

Q42 Solve the equation

$$\frac{4}{2x-1} + 1 = \frac{3}{4x-1}$$

Answer _____ [6]

Q43 A group of *p* people were asked their ages in years.

The total of their ages was 960 years.

When 20 of the people were removed from the group the total of the ages of the remaining group was 480

Given that the mean of the original group was three-quarters of the mean of the remaining group, find the number of people in the original group.

Answer *p* = _____ [3]

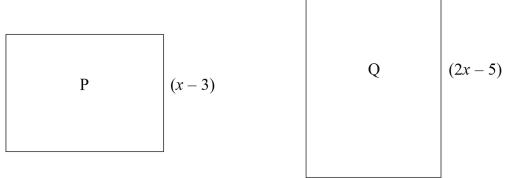
Q44 A group of men and women were asked to give their ages in complete years. The total of the men's ages was 1000 The total of the women's ages was 900 There were 5 more women than men and the mean of the women's ages was 10 smaller than the mean of the men's ages. Find the number of men.

Answer _____ [4]

Q45 Solve
$$\frac{x}{2x-3} + \frac{4}{x+1} = 1$$

Show all your work. A solution by trial and improvement will not be accepted.

Answer *x* = _____ [6]



The width of rectangle P is (x - 3) cm and the width of Q is (2x - 5) cm. The area of rectangle P is 2 cm^2 and the area of rectangle Q is 3 cm^2 . The sum of the lengths of the two rectangles is 3 cm. By forming and solving an equation find the lengths of both rectangles.

Length of rectangle P _____ cm Length of rectangle Q _____ cm [7]

Q47 Solve the equation

$$\frac{3}{3x+5} - \frac{5}{2x+3} = 2$$

Answer [7]

Q48 Solve the following equation giving your answers to 2 decimal places.

$$\frac{x-3}{x+4} - \frac{2x+1}{x+2} = 3$$

Answer _____ [8]

Q49 Simplify

$$\left(\frac{1}{x+4} + \frac{1}{2x-3}\right) \div \frac{3x^2 - 11x - 4}{2x^2 - 32}$$

Answer [6]

Q50

Given that $\frac{2}{x+a} - \frac{5}{x-a} = 1$ has a solution x = 3, find the possible values of a.

Answer [6]

1. (a)	6(2a+1)	A1
(b)	y(y-6)	A1
(c)	b(1+b)	A1

2.	8x - 12 - 2x + 10	MA1
	= 6x - 2	MA1

3.	$6y^2 - 14y - 8y$	MA1 MA1	
	$6y^2 - 22y$	MA1	

4.	4 + 6x - 15 = x + 9	MA1
	6x - x = 9 - 4 + 15	
	5x = 20	MA1
	x = 4	MA1

$$15x - 10 = 7x + 4$$
 MA1

 $15x - 7x = 4 + 10$
 MA1

 $8x = 14$
 MA1

 $x = \frac{14}{8}$ or $1\frac{3}{4}$
 MA1

6. 4c = 2c + 11 MA1 2c = 11 MA1 c = 5.5 A1 22 A1

7.	2x + 10 + 2x + x + 20 = 180	M1	
	5x = 150	MA1	
	x = 30	MA1	
	Smallest angle = 50	A1	

8.	(a)	5t - 10 - 12 + 6t 11t - 22	MA1 A1
	(b)	$e^2 - 7e + 4e - 28$ $e^2 - 3e - 28$	MA1 A1

(a)	8x2 + 6x - 4x - 3 8x2 + 2x - 3 3(8x2 + 2x - 3) = 24x2 + 6x - 9	MA1 MA1 MA1
	Alternative solution	
	(12x+9)(2x-1)24x2+18x-12x-924x2+6x-9	MA1 MA1 MA1
(b)	(t-7)(t+7)	MA1

10. 7x(2xy-5)(A1 for correct partial factorisation)

A2

11.	(a)	$9x^2 - 6xy + y^2$
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(b)
$$(x-5)(x+5)$$

$15w^2 - 24w - 35w + 56$	M1
$15w^2 - 59w + 56$	A1

A2

A1

13.	(a) (i) $3(2a+5)$	A1
	(ii) $x(4-x)$	A1
	(b) $6x + x = 14 + 7$ 7x = 21 x = 3	MA1 MA1 A1
	(c) $\frac{5y}{15} - \frac{3y}{15}$	A1
	$=\frac{2y}{15}$	A1

14.	(a) $2cp(5p-2)$	A1 A1	
	(b) $(y-1)(y+1)$	A1	
	(c) $(k-3)(k+1)$	A2	
	(d) $(x-2)[(x-2)+5]$	A1	
	(x-2)(x+3)	A1	
	alternative solution		
	$(x-2)(x-2) + 5(x-2) = x^2 - 4x + 4 + 5x - 10 = x^2 + x - 6$	A1	
	(x+3)(x-2)	A1	

16.	$p^2 - 6p + 2p - 12$	MA1	
	$p^2 - 4p - 12$	A1	

17.	(a) $p(p-3)$	A1
	(b) $(10-t)(10+t)$	A1
	(c) $(x+5)(x-3)$	A2

18.
 (a)
$$6x^2 + 4x - 15x - 10$$
 MA2
A1

 (b) $\frac{(x+7)(x-7)}{2(x-7)}$
 MA1

 $\frac{x+7}{2}$
 MA1

(a) $(x+7)(x-5)$	MA2
(b) $x = -7 \text{ or } 5$	MA1

20.	x + 9 = 2x + 3 or $x + 9 = 3x - 3$ or $2x + 3 = 3x - 3$	MA1
	appropriate equation solved to get $x = 6$	MA1
	side length $= 15$	MA1
	perimeter $= 45$	A1

3(2x+3) + 4(x-1) = 60	M1
6x + 9 + 4x - 4 = 60	MA1
10x = 55	MA1
x = 5.5 (or equivalent)	A1
	6x + 9 + 4x - 4 = 60 10x = 55

22.	6(x+3) = 2(5x)	MA1
	6x + 18 = 10x	MA1
	18 = 4x	MA1
	$x = 4\frac{1}{2}$	MA1
	alternative solution	
	$\frac{x+3}{2} - \frac{5x}{6} = 0 (\times 6)$	
	3(x+3) - 5x = 0	MA1
	3x + 9 - 5x = 0	MA1
	9 = 2x	MA1
	$x = 4\frac{1}{2}$	A1
	alternative solution	
	$\frac{3(x+3)}{6} = \frac{5x}{6}$	MA1
	3x + 9 = 5x	MA1
	9 = 2x	MA1
	$x = 4\frac{1}{2}$	MA1

23.		
20.	2(a-1) + (a+1) = 12	MA1
	2a - 2 + a + 1 = 12	MA1
	3a = 13	MA1
	$a=4\frac{1}{3}$	MA1

2	4	

multiply by 15	3(2x-1) + 5(4x+5) = 100	MA1
	6x - 3 + 20x + 25 = 100	MA1
	26x = 78	MA1
	x = 3	Al

Alternative Solution:

$\frac{3(2x-1)+5(4x+5)}{15} = \frac{100}{15}$	MA1
6x - 3 + 20x + 25 = 100	MA1
26x = 78	MA1
x = 3	A1

OF	
20	

7y + 19 = 5y + 65	MA1
2y = 46	MA1
<i>y</i> = 23	MA1
$7 \times 23 + 19 = \text{\pounds}1.80 \text{ (or } 5 \times 23 + 65 = \text{\pounds}1.80\text{)}$	A1

do not penalise money notation

y + 3 - 8 = 2(3 - 2y)	MA1
y + 3 - 8 = 6 - 4y	MA1
5y = 11	MA1
$y = \frac{11}{5} = 2\frac{1}{5}$	MA1
Alternative solution	
Alternative solution $\frac{1}{4}y + \frac{3}{4} - 2 = \frac{3}{2} - y$	MA1
	MA1 MA1
$\frac{1}{4}y + \frac{3}{4} - 2 = \frac{3}{2} - y$	

27.	$\frac{5t-3t}{6}$ (or equivalent fractions with different denominator)	MA1
	$=\frac{2t}{6}$	A1
	$=\frac{t}{3}$	A1

$\frac{2(2x-1)+3(x+2)+x}{6} = 8$	MA1
$\frac{4x - 2 + 3x + 6 + x}{6} = 8$	MA1
$\frac{8x+4}{6} = 8$	MA1
8x = 44	MA1
$x = 5\frac{1}{2}$	A1
alternative solution	
2(2x-1) + 3(x+2) + x = 48	MA2
4x - 2 + 3x + 6 + x = 48	MA1
8x = 44	MA1
$x = 5\frac{1}{2}$	A1
	$\frac{4x - 2 + 3x + 6 + x}{6} = 8$ $\frac{8x + 4}{6} = 8$ 8x = 44 $x = 5\frac{1}{2}$ alternative solution 2(2x - 1) + 3(x + 2) + x = 48 4x - 2 + 3x + 6 + x = 48 8x = 44

29.	6(3x+2) - 5(3x-4) = 2	MA1
	18x + 12 - 15x + 20 = 2	MA1
	3x = -30	MA1
	x = -10	Al

Alternative Solution:

$\frac{6x}{5} + \frac{4}{5} - x + \frac{4}{3} = \frac{2}{15}$	MA2
$\frac{x}{5} = -2$	MA1
x = -10	A1

follow for numerical errors, but not omission of key elements of method

30.
$$\frac{x(x-2)}{3} \times \frac{6}{(x+4)(x-2)}$$
 MA2
= $\frac{2x}{x+4}$ A1

31.
$$\frac{10 - (x + 10)}{2}$$
 MA1
= $-\frac{x}{2}$ A1
Alternative solution
 $5 - \frac{x}{2} - \frac{10}{2}$ MA1
= $-\frac{x}{2}$ A1

$$\frac{x^2(x-3)}{(x+3)(x-3)} = \frac{x^2}{x+3}$$
 MA1 MA1 MA1

$5a(20x^2 + 13xy - 15y^2)$	A1
= 5a(5x-3y)(4x+5y)	A1 A1

34.	(a)	$20x^2 - 15x + 8x - 6$ $20x^2 - 7x - 6$	MA1 MA1
	(b)	$\frac{x(x+4)}{(x+4)(x-4)}$	MA1 MA1
		$\frac{x}{x-4}$	MA1
	(c)	$a(8x^2 - 26xy + 15y^2) a(4x-3y)(2x-5y)$	MA1 MA2

35.	(3x-4y)(2x-3y)	MA2

36.	$\frac{2(3a-4)(a+2)}{2(2-a)(2+a)}$	MA1 MA1	
	$\frac{(3a-4)}{(2-a)}$	A1	

(a) $5a (6x^2 + xy - 12y^2)$	MA1
5a (3x - 4y)(2x + 3y)	MA1 MA1
(b) $x = \frac{4y}{3}$ $x = \frac{-3y}{2}$	A1 A1

38.	$\frac{3x(x-2y)}{(4x-3y)(x-2y)}$	MA1
	Answer = $3x / (4x - 3y)$	MA1
39.	(3x-2y)(5x+4y)	MA2

$2(4x^2-25y^2)$	MA1
2(2x-5y)(2x+5y)	MA2

41. (a)
$$(2a-b)(a+4b)$$
 M1 A1
(b) $\frac{(x+1)(x-4)+(3x-4)(2x-1)}{(2x-1)(x-4)} \times \frac{2x-1}{x}$ MA1
 $\frac{x^2-3x-4+6x^2-11x+4}{(2x-1)(x-4)} \times \frac{(2x-1)}{x}$ MA1
 $\frac{7x^2-14x}{(x-4)} \times \frac{1}{x}$ MA1
 $\frac{7(x-2)}{x-4}$ or $\frac{7x-14}{x-4}$ A1

A1

42.

$$4(4x-1) + (2x-1)(4x-1) = 3(2x-1)$$
MA2

$$16x - 4 + 8x^{2} - 2x - 4x + 1 = 6x - 3$$

$$8x^{2} + 4x = 0$$
MA1

$$4x(2x+1) = 0$$
MA1
So $x = 0$ or $x = -\frac{1}{2}$
A1 A1

43. $\frac{960}{p} = \frac{3}{4} \times \frac{480}{p - 20}$ M1 8p - 160 = 3p MA1 p = 32 A1

MA2
MA1
A1

45. $\frac{x(x+1) + 4(2x-3)}{(2x-3)(x+1)} = 1$	M1
$\frac{x^2 + 9x - 12}{2x^2 - x - 3} = 1$	MA1
$2x^2 - x - 3 = x^2 + 9x - 12$	MA1
$x^2 - 10x + 9 = 0$	MA1
(x-1)(x-9) = 0	MA1
x = 1, x = 9	A1

$$\frac{2}{x-3} + \frac{3}{2x-5} = 3$$
MA1

$$2(2x-5) + 3(x-3) = 3(x-3)(2x-5)$$
MA1

$$4x - 10 + 3x - 9 = 3[2x^2 - 6x - 5x + 15]$$
MA1

$$7x - 19 = 6x^2 - 33x + 45$$

$$6x^2 - 40x + 64 = 0$$
MA1

$$3x^2 - 20x + 32 = 0$$
(3x - 8)(x - 4) = 0
MA1

$$x = 4, x = 8/3 \text{ (invalid)}$$
A1
length P = 2, length Q = 1
A1

47.

$$3(2x+3) - 5(3x+5) = 2(3x+5)(2x+3)$$

$$6x+9 - 15x - 25 = 2(6x^{2} + 19x + 15)$$

$$6x+9 - 15x - 25 = 12x^{2} + 38x + 30$$
MA1

$$12x^{2} + 47x + 46 = 0$$
MA1

$$(x+2)(12x+23) = 0$$
MA1

$$x = -2 \text{ or } x = -\frac{23}{12}$$
A1 A1

$$\frac{(x-3)(x+2) - (2x+1)(x+4)}{(x+4)(x+2)} = 3$$
MA1
$$\frac{x^2 - x - 6 - 2x^2 - 9x - 4}{x^2 + 6x + 8} = 3$$
MA2
$$-x^2 - 10x - 10 = 3x^2 + 18x + 24$$
MA1
$$4x^2 + 28x + 34 = 0$$
MA1
$$x = \frac{-28 \pm \sqrt{28^2 - 4 \times 4 \times 34}}{8}$$
M1
$$x = -1.56, -5.44$$
MA1

23
$$\left(\frac{(2x-3)+(x+4)}{(x+4)(2x-3)}\right) \times \frac{2x^2-32}{(3x^2-11x-4)}$$
 MA2
 $\frac{3x+1}{(x+4)(2x-3)} \times \frac{2(x-4)(x+4)}{(x-4)(3x+1)}$ MA3
 $\frac{2}{(2x-3)}$ MA1

$$\frac{2}{3+a} - \frac{5}{3-a} = 1$$
MA1

$$\frac{2(3-a) - 5(3+a)}{(3+a)(3-a)} = 1$$
MA1

$$6 - 2a - 15 - 5a = 9 - a^2$$
MA1

$$a^2 - 7a - 18 = 0$$
MA1

$$(a - 9)(a + 2) = 0$$
MA1

$$a = 9 \text{ or } -2$$
A1