St. Patrick's High School, Keady

Mathematics Department

GCSE Mathematics Practice Booklet

M4

Topic 5 – Geometry and Measures 2

Angles

Circle Theorems

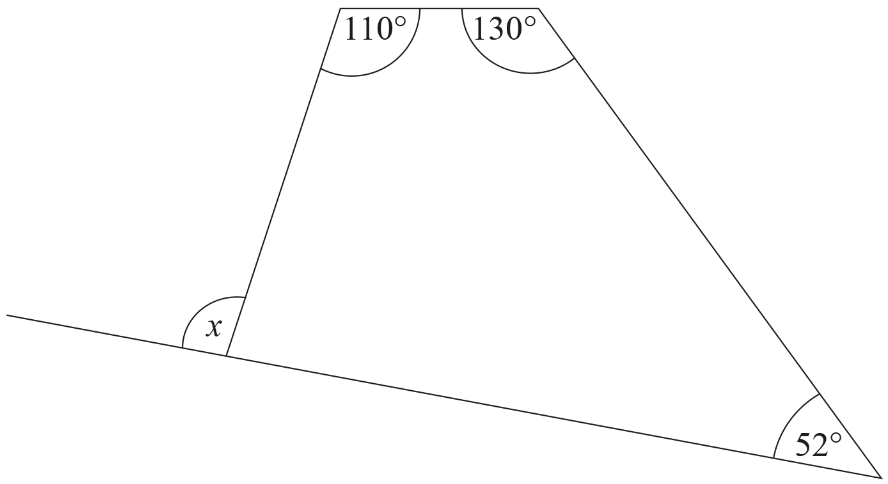
Questions taken from CCEA Past Papers

Mark Scheme included at the end of this booklet

Keady Maths

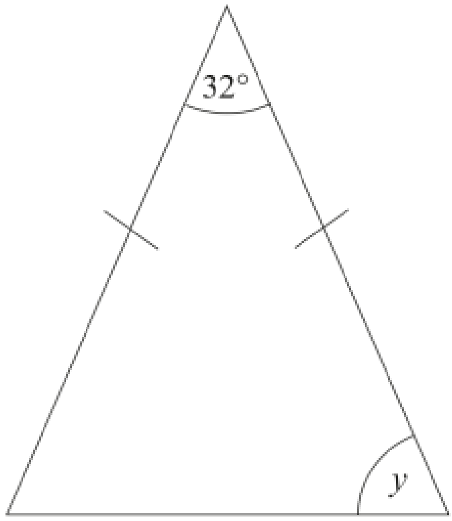
Q1

(a) Work out the size of the angle x in the diagram below.



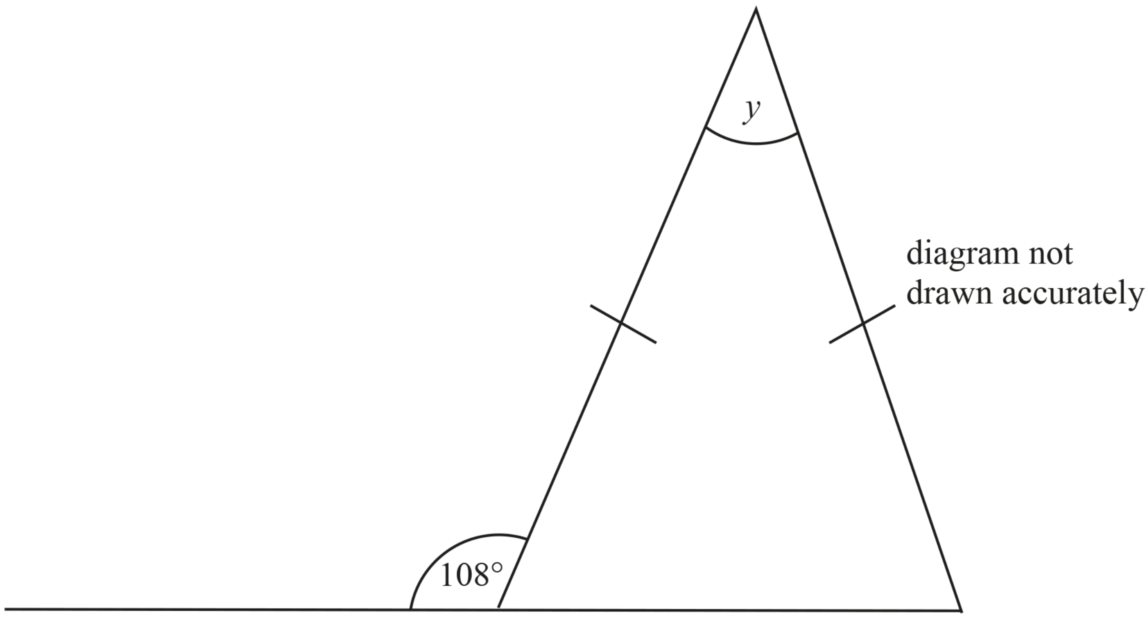
Answer ____________° [3]

(b) Work out the size of the angle y in the diagram below.



Answer ____________° [2]

Q2 Work out the size of angle y in the diagram below.



Answer y = _____________° [3]

Q3 Work out the size of the angle w.

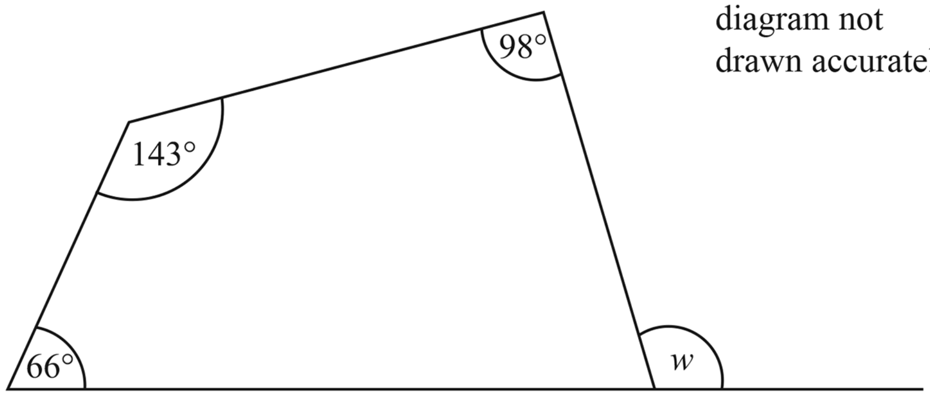


diagram not drawn accurately

Answer w = ________________________ ° [3]

Q4 The triangle shown is isosceles. ABC is a straight line.

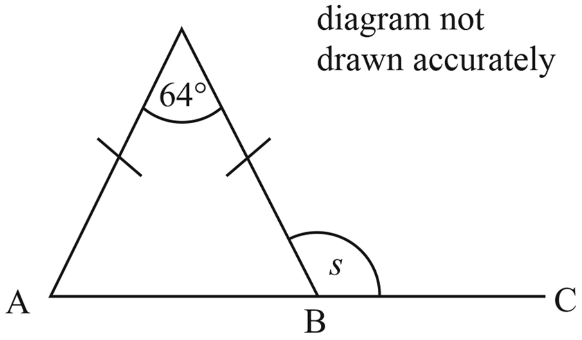


diagram not drawn accurately

Work out the size of the angle *s*.

Answer $s =$ ________________ ° [3]

Q5 Calculate the size of angle x in this quadrilateral.

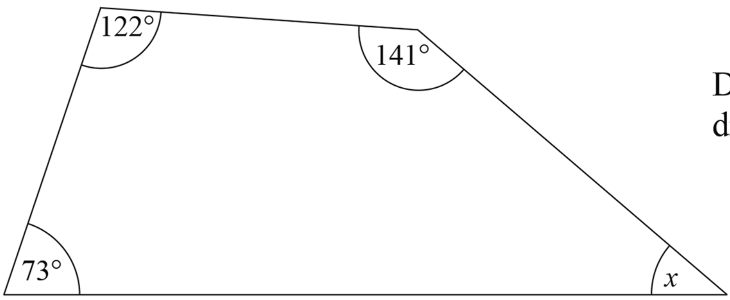
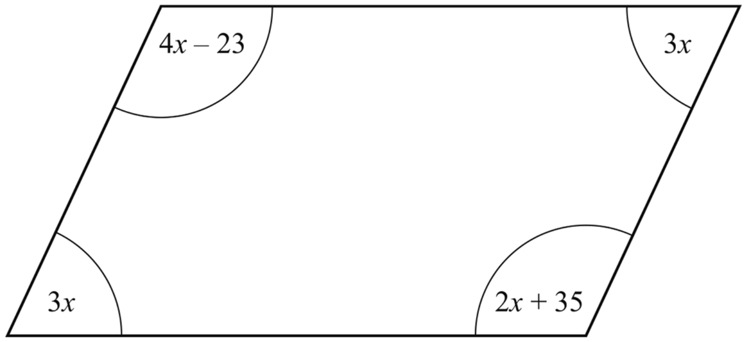


Diagram not drawn accurately

Answer $x =$ __________ ° [2]

Q6



The diagram above is a parallelogram.
The sizes of the angles in degrees are $3x$, $4x - 23$, $3x$ and $2x + 35$
Work out the value of x.

Answer x = ______________ [3]

Q7

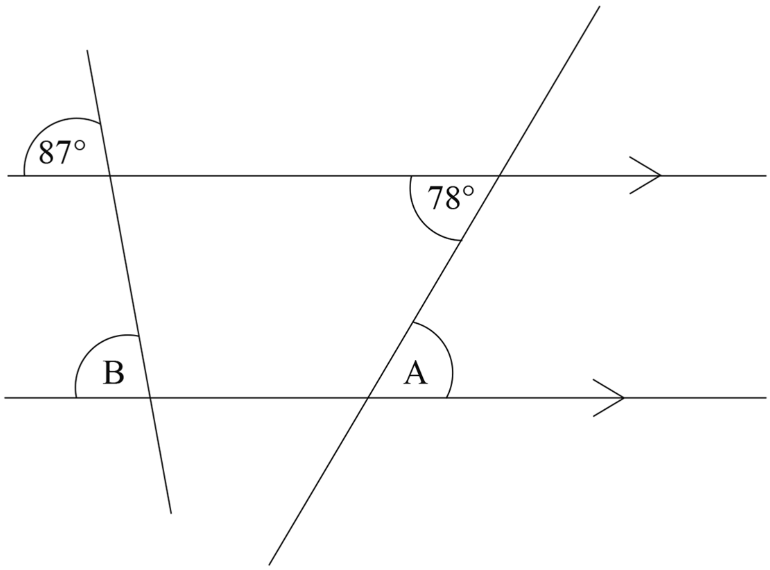


diagram
not
drawn
accurately

Find the size of angle

(a) A Answer _____________° [1]

(b) B Answer _____________° [1]

Q8

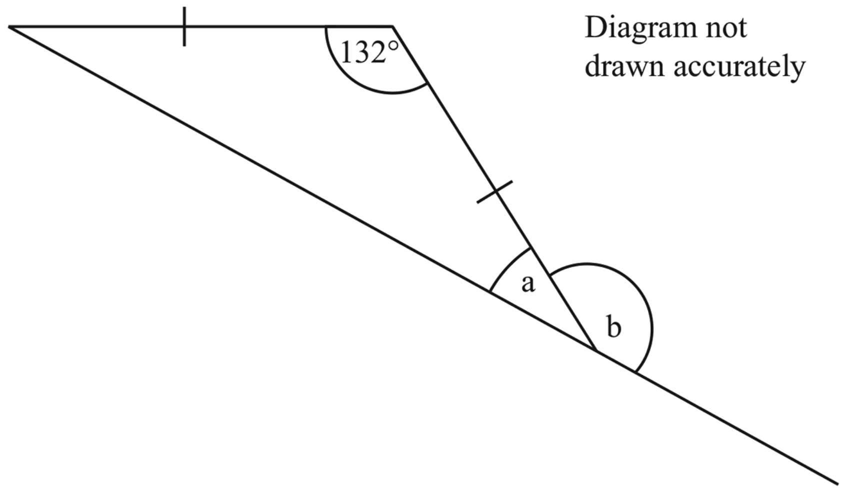


Diagram not drawn accurately

Find the size of

(a) angle a

Answer a = ________° [2]

(b) angle b

Answer b = ________° [1]

Q9

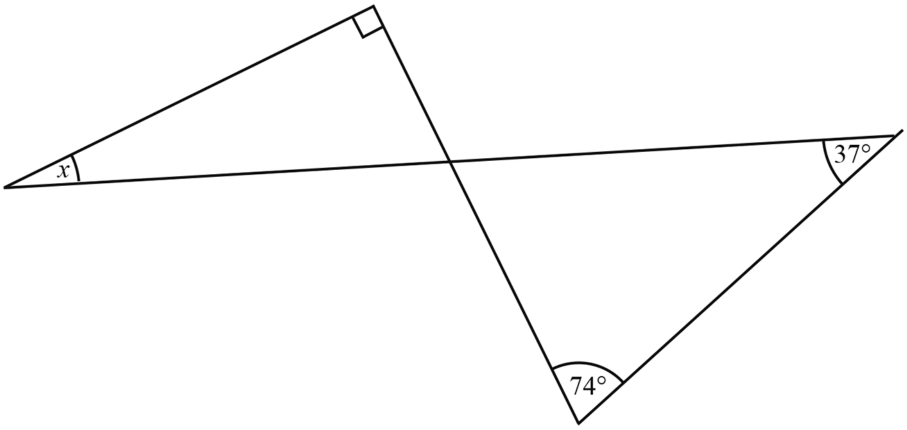


diagram not drawn accurately

Calculate the size of the angle marked x.

Answer _____________ ° [3]

Q10 In the diagram lines AB and CD are parallel.

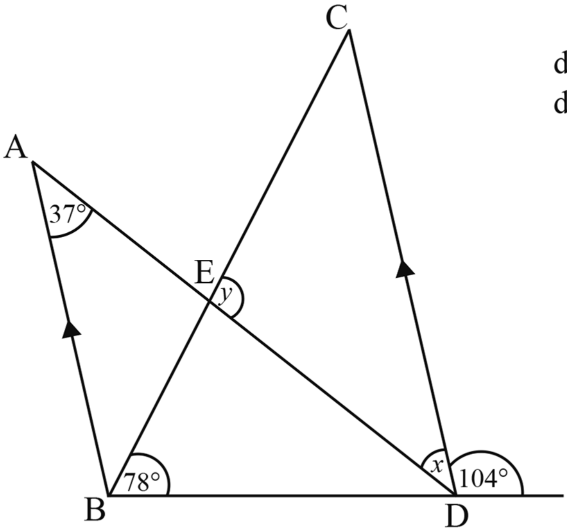


diagram not drawn accurately

(a) Find the size of the angle *x*.

Answer _____________° [1]

(b) Calculate the size of the angle *y*.

Answer _____________° [2]

Q11

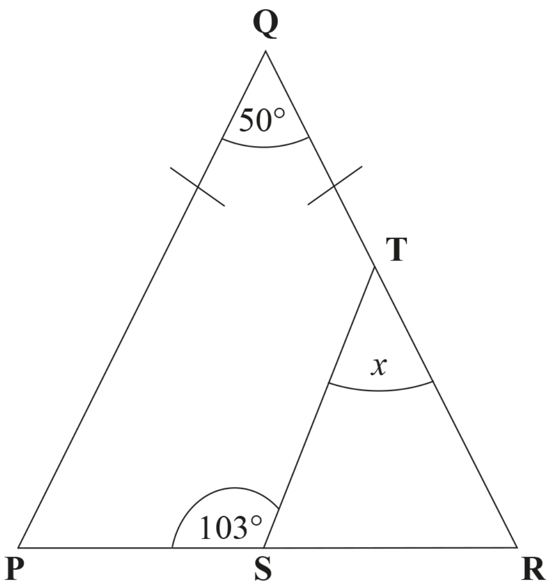


diagram
not drawn
accurately

Triangle PQR is isosceles with PQ = QR.

(a) Calculate the size of angle x

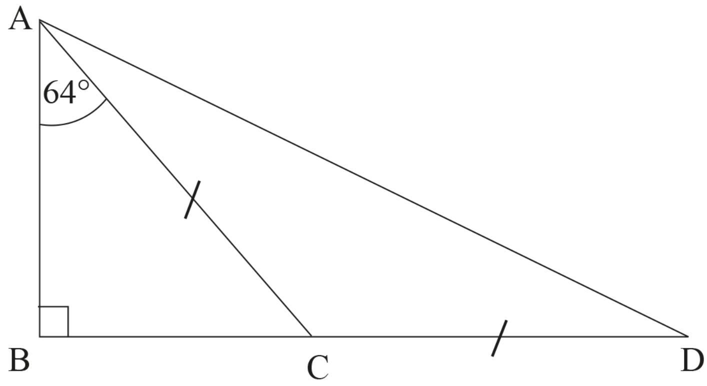
Answer _____________° [3]

(b) Hence decide if the lines PQ and ST are parallel.

_________ because ____

____ [2]

Q12



diagram
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drawn
accurately

ABC is a right-angled triangle.
ACD is an isosceles triangle.
BCD is a straight line.

Calculate the size of

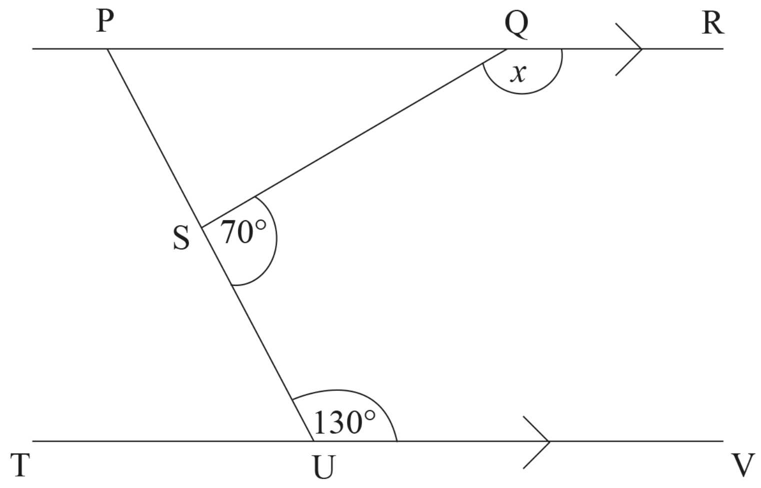
(a) angle ACB,

Answer _____________ ° [2]

(b) angle ADC.

Answer _____________ ° [3]

Q13



diagram
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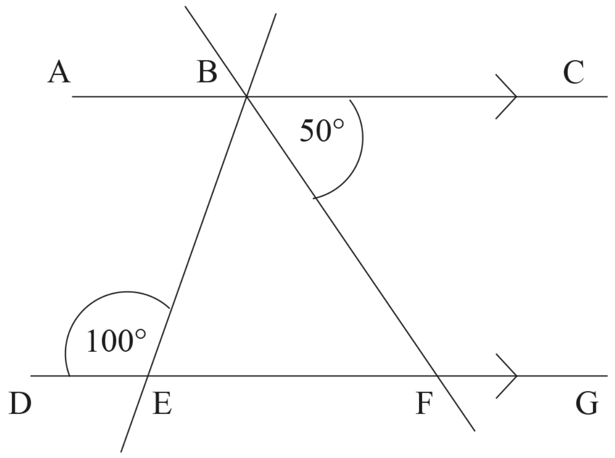
accurately

PR and TV are parallel lines.

Calculate the size of angle x.

Answer __________° [3]

Q14



diagram
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drawn
accurately

AC and DG are parallel lines.

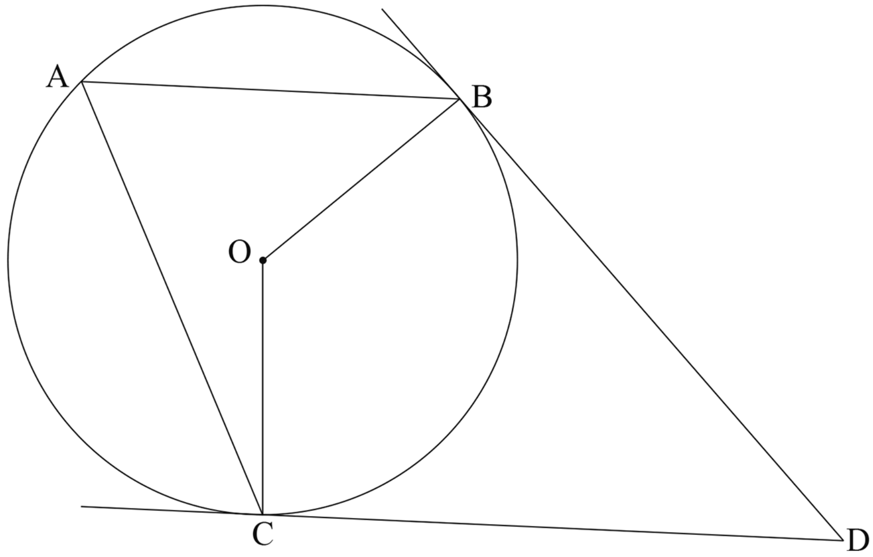
Angle CBF = 50° and angle BED = 100°

What type of triangle is BEF?

Give a reason for each angle found.

Answer ______________________ [3]

Q15



O is the centre of the circle. A, B, C are points on the circumference.

DB and DC are tangents. The angle CAB is x.

Find, in simplest form, in terms of x,

(a) angle BOC,

Answer ____________________ [1]

(b) angle BDC.

Answer ____________________ [2]

Given that the lines AB and CD are parallel find, in simplest form, in terms of x,

(c) angle ABO,

Answer ____________________ [2]

(d) angle ACO.

Answer ____________________ [2]

Q16 Prove that BC is parallel to the tangent TU in the diagram shown.

Justify each step of your proof.

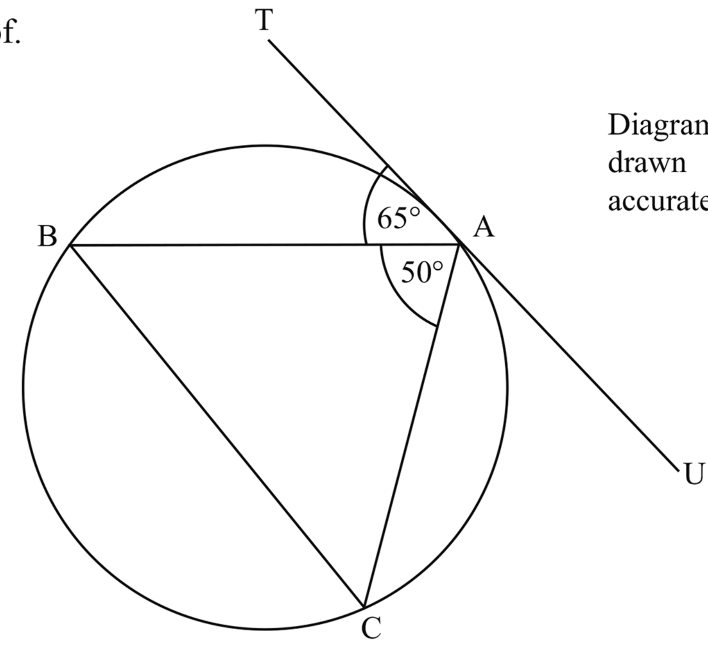
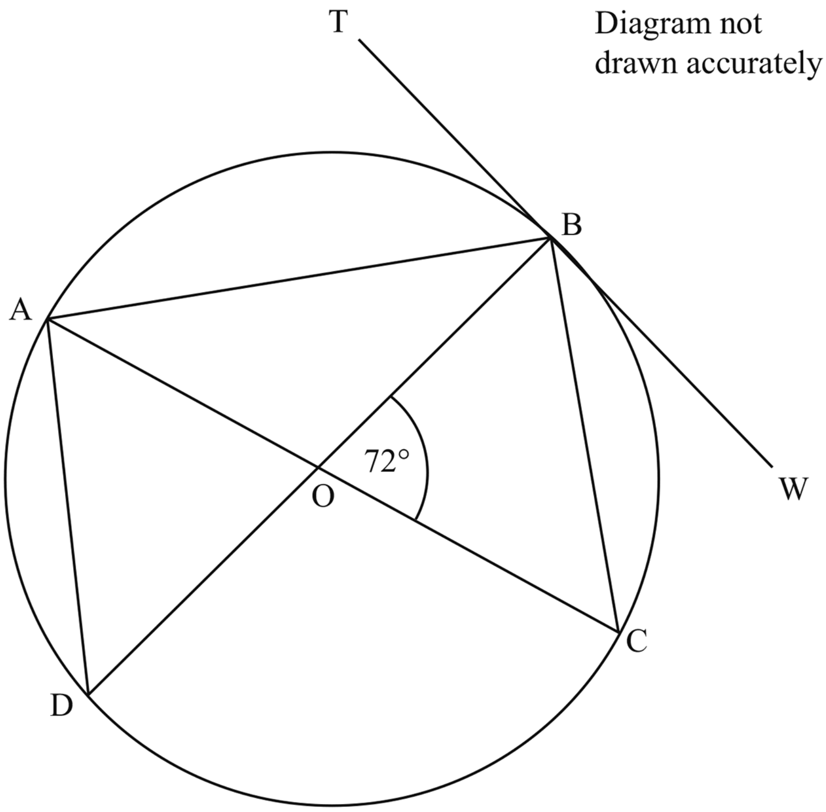


Diagram not drawn accurately

[3]

Q17



O is the centre of the circle and the tangent TW touches the circle at B.

Find the size of the angles

(a) TBO

Answer ________________° [1]

(b) CAB

Answer ________________° [1]

(c) CBW

Answer ________________° [1]

(d) DBC

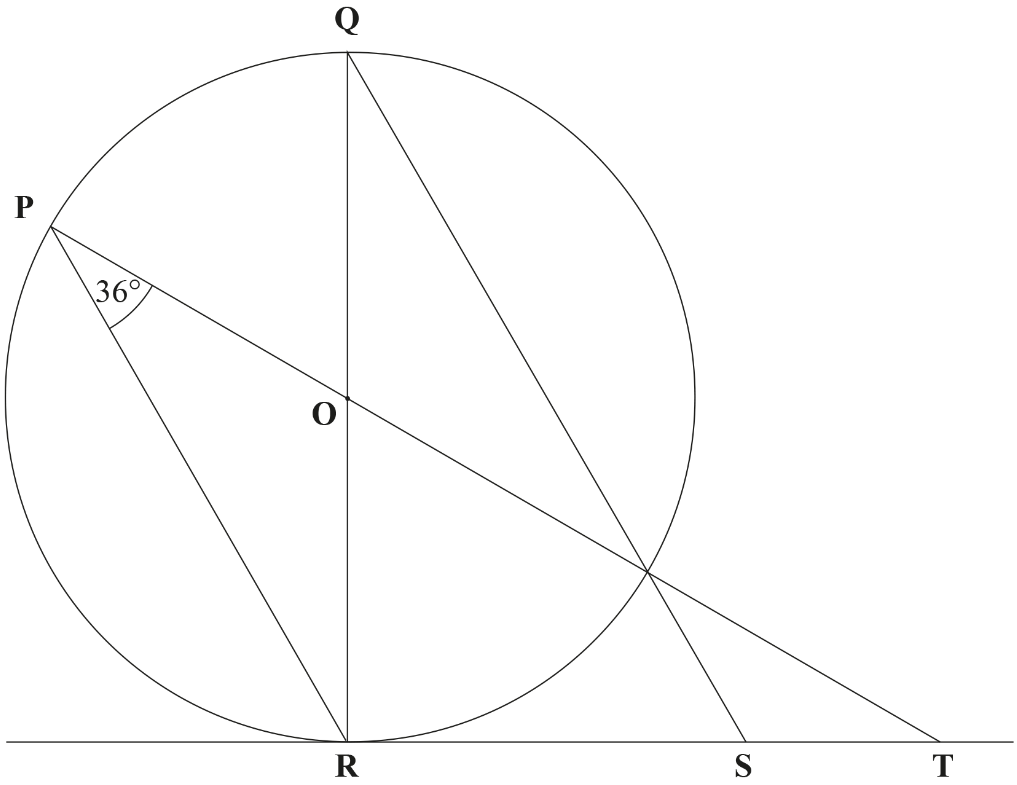
Answer ________________° [1]

Q18

(a) In the diagram shown, O is the centre of the circle.

P, Q and R are points on the circumference of the circle.

RST is a tangent to the circle.



Calculate the size of

(i) angle ROT,

Answer _____________° [1]

(ii) angle OTR,

Answer _____________° [1]

(iii) angle QSR.

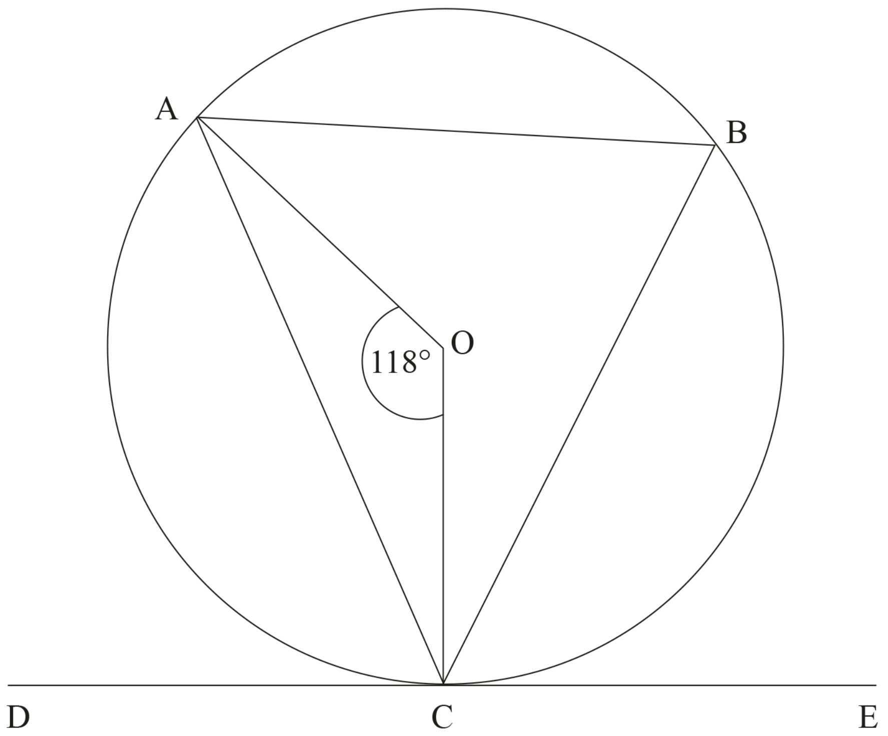
Answer _____________° [1]

Q19

(b) O is the centre of the circle through A, B and C.

DCE is a tangent to the circle at C.

Angle AOC = 118°



(i) Find the size of angle ACD.
Give a reason for each step.

Answer Angle ACD = _____________°

Reasons:

[3]

(ii) Angle BCE = x.

Express the angles BAC and BCA in terms of x.

Answer BAC = ______________° [1]

BCA = ______________° [2]

Q20

A, B, C and D are points on the circumference of a circle with centre M.

BD is the diameter of the circle.

Angle BAC = 38°

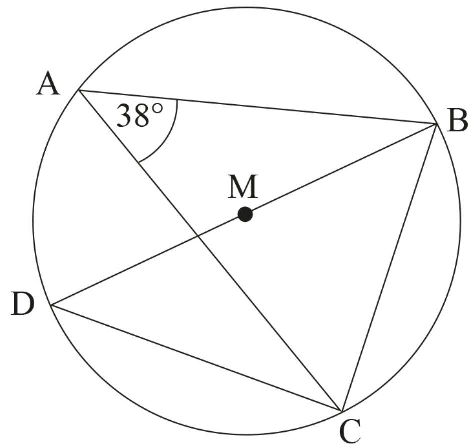


diagram
not drawn
accurately

(a) Find the size of angle BDC, giving a reason for your answer.

Answer _________° because ____ [2]

(b) Find the size of angle BCD, giving a reason for your answer.

Answer _________° because ____ [2]

(c) Find the size of angle BMC, giving a reason for your answer.

Answer _________° because ____ [2]

Q21

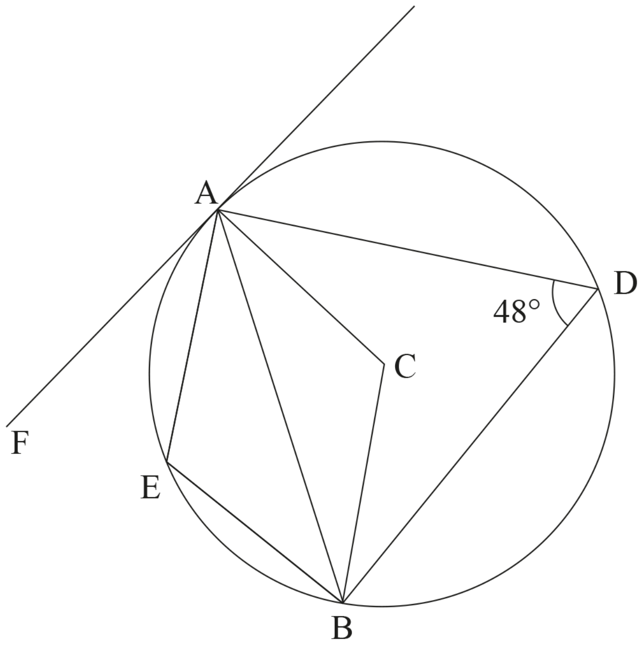


diagram not
drawn accurately

A, B, D and E are points on the circumference of a circle, centre C.

$\angle ADB = 48°$

AF is a tangent to the circle.

Find the size of the following angles, giving a reason for each answer.

(a) $\angle$ AEB = _________ ° because ____

____ [2]

(b) $\angle$ ACB = _________ ° because ____

____ [2]

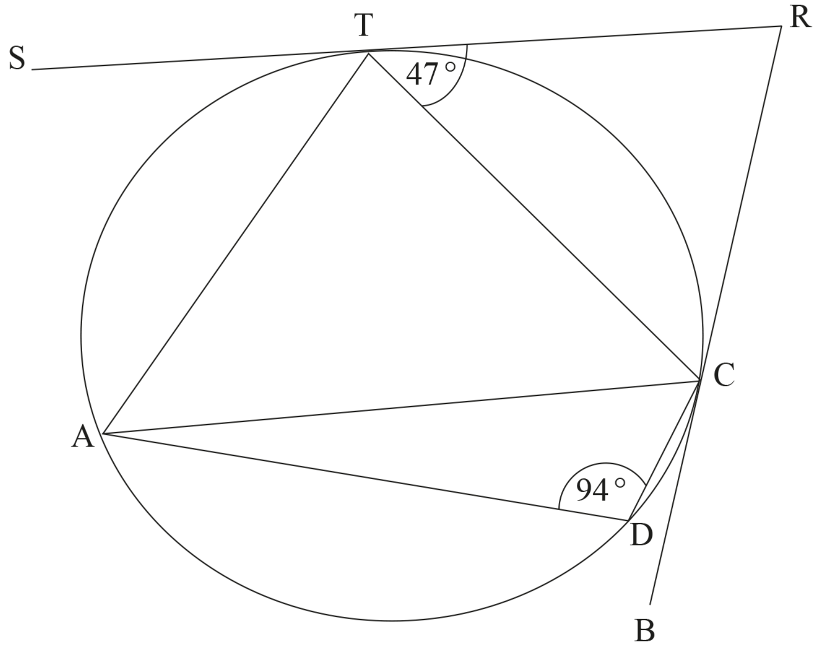
(c) $\angle$ BAF = _________ ° because ____

____ [2]

Q22

The lines STR and BCR are tangents to the circle shown.

Angle RTC = 47° and angle ADC = 94°



diagram
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accurately

John proved that the lines AC and SR are parallel.
He used the following proof but didn't give his reasons.

Using the properties of tangents and circle theorems complete John's argument.

1. Angle RCT = 47° because ____

2. Angle RTC = Angle TAC because ______________________________________

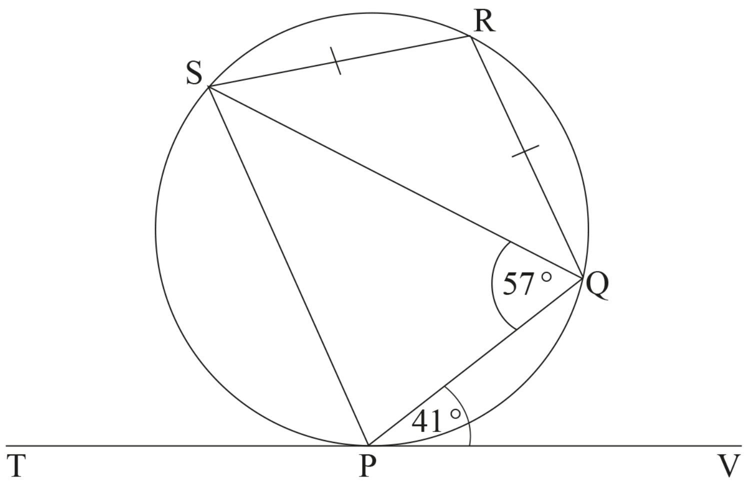
3. Angle ATC = 86° because ____

4. Angle STA = 47° because ____

5. So the lines AC and SR are parallel because ____________________________

[5]

Q23



diagram
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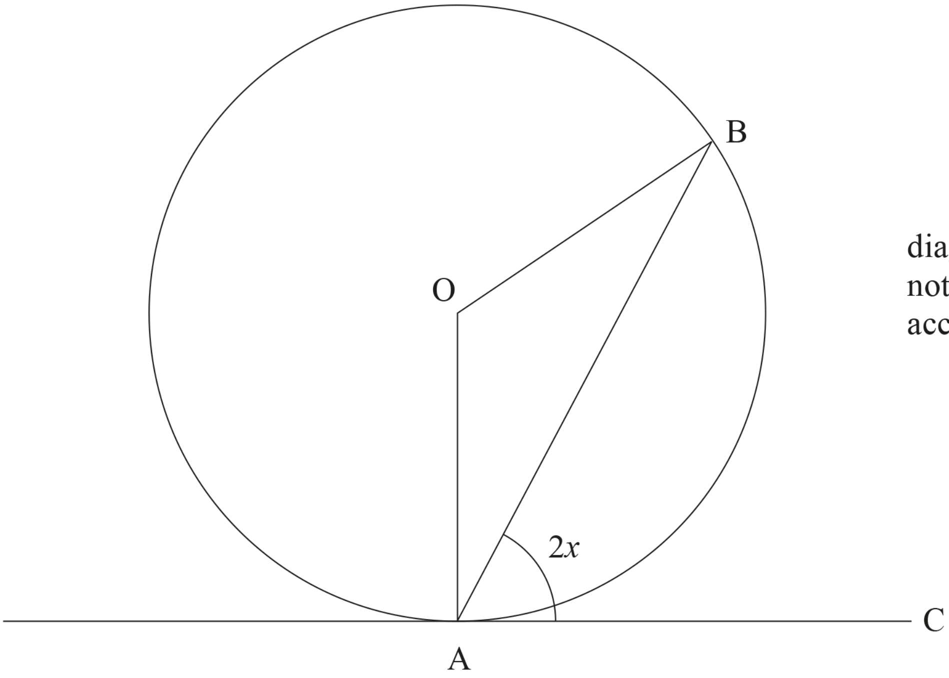
accurately

TV is a tangent to the circle at P.
SR = RQ
Angle QPV = 41° and angle SQP = 57°
Show that SP is parallel to RQ.
You must give reasons to justify any angles that you calculate.

[5]

Q24



diagram
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accurately

A and B are points on the circumference of a circle, centre O.

AC is a tangent to the circle.

Angle BAC = $2x$

Find the size of the angle AOB, in terms of x, giving a reason for each stage of your working.

Answer ______________ ° [3]

Q25 In the diagram shown

QR = RS

PQ is parallel to SR

TPV is a tangent to the circle at the point P

Angle SPV = 52°

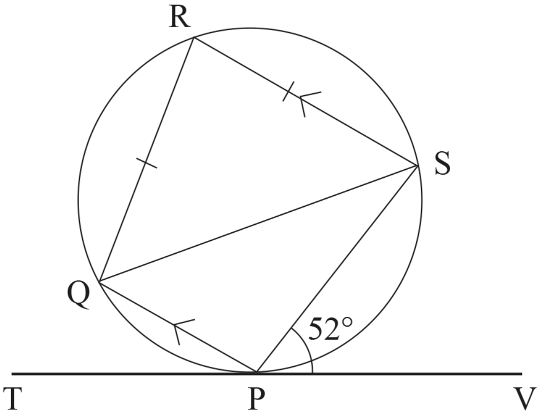


diagram
not drawn
accurately

Find the size of angle SPQ explaining clearly each step of your solution.

Answer Angle SPQ = _____________° [6]

Q26

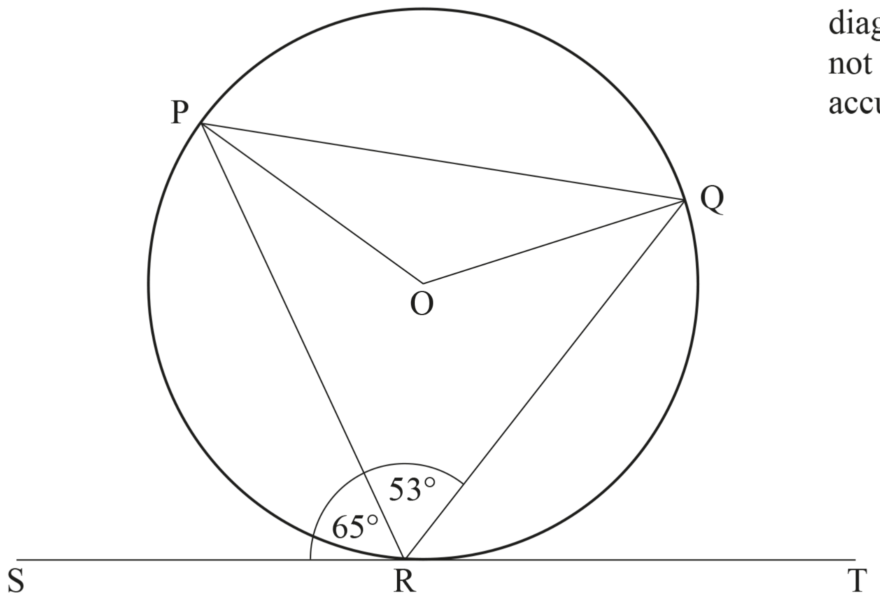


diagram
not drawn
accurately

P, Q and R are points on the circumference of a circle, centre O.
The line ST is a tangent to the circle.
Angle PRS = 65°
Angle PRQ = 53°

(a) Calculate the size of angle POQ, giving a reason for your answer.

Answer _____________ ° because ______________________________________

____ [2]

(b) Calculate the size of angle OQR, giving reasons for each step of your answer.

Answer _____________ ° [3]

Q27 **(a)** O is the centre of the circle.

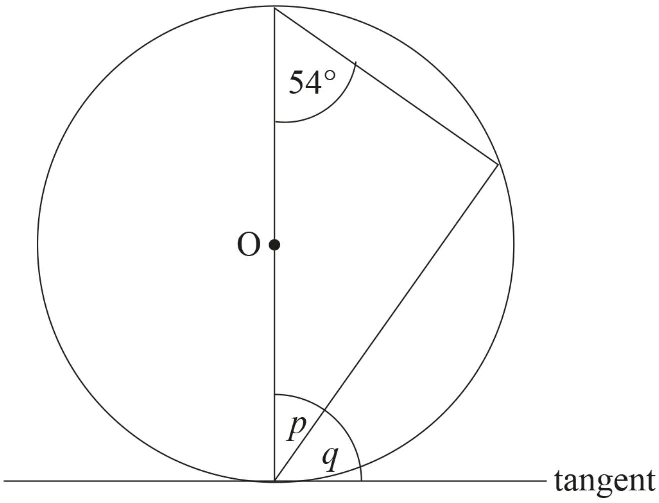


diagram not drawn accurately

Calculate the size of angle

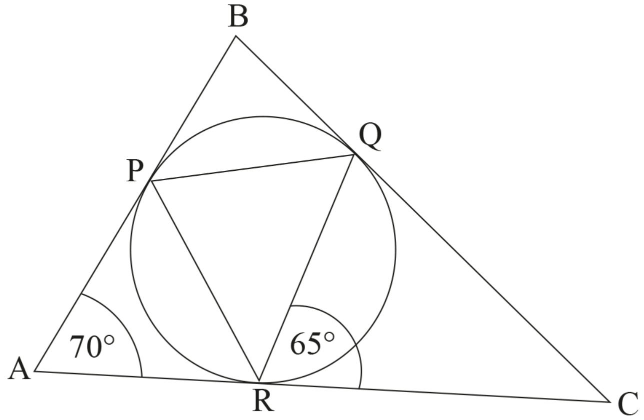
(i) p

Answer _________° [1]

(ii) q

Answer _________° [1]

(b) The lines AB, BC, CA are tangents to the circle.



diagram
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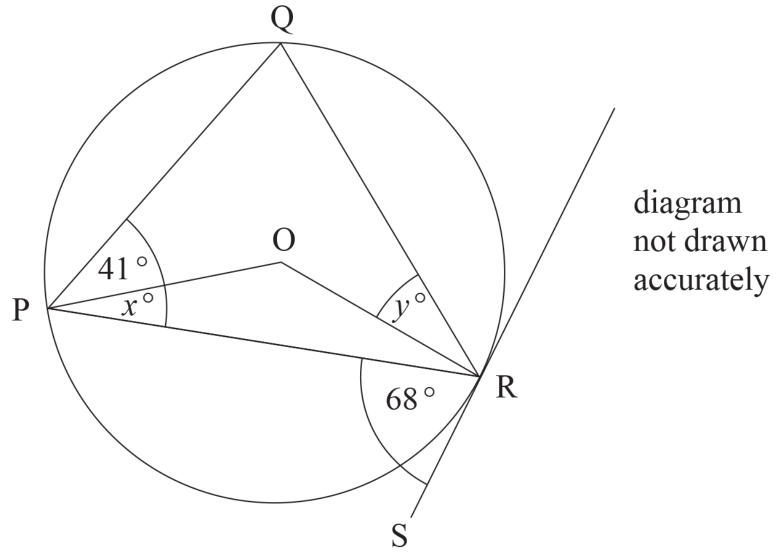
What type of triangle is triangle **BPQ**?

You must explain your reasoning clearly.

Answer ______________________ [4]

Q28



The diagram shows a circle with centre O.

SR is a tangent to the circle.

Find the size of angles x and y giving reasons for each stage of your working.

Answer $x =$ ____________ °

$y =$ ____________ ° [6]

1.	**(a)** $360 - (110 + 130 + 52)$	MA1
	68	MA1
	112	A1
	(b) $(180 - 32) \div 2$	MA1
	74	A1

2.	$180 - 108 = 72$	MA1
	$180 - (72 \times 2)$	MA1
	36	A1

3.	$360 - (66 + 143 + 98)$ or $360 - 307$	MA1
	53	A1
	127	MA1

4.

$(180 - 64) \div 2$	M1
58	A1
122	MA1

5.

$360 - (122 + 141 + 73)$ or $360 - 336$	M1
24	A1

6.

$4x - 23 = 2x + 35$	**or**	$3x + 4x - 23 + 3x + 2x + 35 = 360$	M1
$2x = 58$	**or**	$12x = 348$	MA1
$x = 29$		$x = 29$	MA1

or

$4x - 23 + 3x = 180$	**or**	$2x + 35 + 3x = 180$	M1
$7x = 203$	**or**	$5x = 145$	MA1
$x = 29$		$x = 29$	MA1

7.

(a) 78°	A1
(b) 87°	A1

8. **(a)** $180 - 132 = 48$ M1

$48 \div 2$

24 A1

(b) $180 - (a) = 156$ MA1

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9. $180 - (74 + 37) = 69$ MA1

vertically opposite angle = 69 A1

$x = 180 - (90 + 69) = 21$ MA1

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10. **(a)** 37° A1

(b) ABE = 104 – 78 = 26° MA1

AEB = 180 – (26 + 37) = 117° = y MA1

or

EDB = 180 – (104 + 37) = 39°

BED = 180 – (78 + 39) = 63° MA1

y = 180 – 63 = 117° MA1

or

BDC = 76°

BCD = 180 – (76 + 78) = 26° MA1

y = 180 – (26 + 37) = 117° MA1

-

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11. **(a)** QPR = QRS = 65° (mark gained for angle QRS as 65 in diagram) MA1
TSR = 77° (may be marked in diagram) MA1
x = 180 – (77 + 65) = 38° (3 marks for correct ans) MA1

(b) No because 50 + 142 ≠ 180°
or because 65 + 103 ≠ 180°
or because the angles between the two lines do not add up to 180 so not parallel
or because 38 ≠ 50, corresponding.
Allow A1 for numerical error but correct argument A2

12.

(a) 180 – 90 – 64 **or** 90 – 64 M1
= 26 A1

(b) 180 – 26 = 154 MA1

$\frac{180 - 154}{2}$ M1

= 13 A1

13.

QSP = 110 MA1

TUP = 50 so QPU = 50 (alternate) MA1

PQS = 180 – (50 + 110) = 20, x = 180 – 20 = 160 MA1

14.

angle BFE = 50, alternate MA1

angle BEF = 80, angles on straight line add to 180° MA1

angle EBF = 50, angle sum of triangle, so triangle is isosceles MA1

15.

(a) $2x$ A1

(b) Uses fact that tangent angles are $90°$ M1

$180 - 2x$ MA1

(c) Angle ABD M1

$2x - 90$ MA1

(d) $360 - 2x$ seen or angle ACT = $x°$ M1

$90 - x$ MA1

16.

ACB = 65° AST C1
ABC = 65° Angles of a triangle C1
So TU is parallel to BC, alternate angles equal C1

17.

(a) 90 A1

(b) 36 A1

(c) 36 A1

(d) 54 A1

18.

(a) **(i)** Angle ROT = 72° A1

(ii) Angle OTR = 18° A1

(iii) Angle QSR = 54 A1

19.

(b) **(i)** ACD = 59° A1

Because ABC = 59° – angle at centre is twice angle on circumference
(allow angle at circumference is half the angle at the centre) MA1
So ACD = 59° – angle in alternate segment MA1

alternative solution

AOC is isosceles so OCA = 31° MA1

OCD is 90° MA1

ACD = 90 – 31 = 59° A1

(ii) BAC = x A1

BCA = 180° – (x + 59°) MA1

= 121° – x A1

20.

(a) 38° A1
Because angles in the same segment are equal MA1

(b) 90° A1
Because the angle in a semi-circle is 90° MA1

(c) 76° A1
Because the angle at the centre is twice the angle at the circumference MA1

21.

(a) 132° because opposite angles in a cyclic quadrilateral add up to 180° A1 A1

(b) 96° (or 264°) because the angle at the centre is twice the angle on the circumference A1 A1

(c) 48° because of the Alternate Segment Theorem (or 90 − 42 = 48° using tangent/radius and angles in isosceles triangle ABC) A1 A1

22.

1. TRC is isosceles, tangents are the same length A1
2. Alternate Segment Theorem A1
3. Opposite Angles of Cyclic Quad add to give 180° A1
4. Angles on a straight line add to 180° A1
5. Angle STA = Angle TAC and are alternate A1

23.

Angle PSQ = 41° (alternate segment theorem)	MA1
Angle SPQ = 82° (180 – 57 – 41 triangle)	MA1
(**or** Angle SPT = 57° (alternate segment theorem)	MA1
and Angle SPQ = 82° (straight line)	MA1)
Angle SRQ = 98° (opposite angle in cyclic quadrilateral)	MA1
Angle SQR = 41° (isosceles triangle)	MA1
So SP parallel to RQ as alternate angles PSQ and SQR are equal	MA1

24.

Using Alternate Segment Theorem angle on circumference is $2x$	MA2
Angle at centre is double the angle on the circumference so AOB = $4x$	MA1

alternative solution

<OAB = $90 - 2x$	angle between tangent and radius is 90°	MA1
<ABO = $90 - 2x$	angles in isosceles triangle equal	MA1
<AOB = $180 - (90 - 2x) - (90 - 2x) = 4x$	angles in triangle add to 180°	MA1

25.

Angle PQS = 52° because AST	MA1MA1
Angle RSQ = 52° because alternate angles are equal	MA1
Angle SRQ = 76° because isosceles triangle	MA1
Angle SPQ = 104° because opposite angles in a cyclic quadrilateral add up to 180°	MA1MA1

26.

(a) Angle POQ = 106° because the angle at the centre is twice the angle at the circumference — A2

(b) Angle PQR = 65° Alternate Segment Theorem — MA1

Angle PQO = 37° isosceles triangle (radii equal) — MA1

Angle OQR = 65 – 37 = 28° — MA1

27.

(a) **(i)** p = 36° — A1

(ii) q = 54° — A1

(b) ARP = 55, two tangents equal so isosceles triangle — MA1

PRQ = 60, adjacent or straight line — MA1

BPQ = 60, alternate segment — MA1

BQP = 60, 2 tangents; isosceles or alternate segment theorem so PBQ = 60, angles in triangle : equilateral — MA1

alternative solution

RCQ = 50, two tangents equal so isosceles triangle — MA1

ABC = 60, angles in a triangle — MA1

BPQ = BQP = 60, 2 tangents so isosceles triangle or alternate segment theorem — MA1

Equilateral — A1

28.

Angle PQR = 68°	Alternate Segment Theorem	A1 MA1
Angle POR = 136°	Angle at centre is twice the angle at the circumference	A1 MA1
$x = 22°$	Isosceles triangle	MA1
$y = 27°$	Angles in quadrilateral add up to 360°	MA1

Alternative Solution

Angle PRO = 22°	Tangent/radius 90°	MA1
$x = 22°$	Isosceles triangle	MA1
Angle POR = 136°		A1
Angle PQR = 68°	Angle at centre = 2 × angle at circumference	A1 MA1
$y = 27°$	Angles in quadrilateral add up to 360°	MA1