St. Patrick's High School, Keady

Mathematics Department

GCSE Mathematics Practice Booklet

M4

Topic 9 –Geometry and Measures 3

Arcs, Sectors, Cylinders, Cones and Spheres

Compound Measures

Questions taken from CCEA Past Papers

Mark Scheme included at the end of this booklet

Q1 A lorry travels 240 km in 150 minutes.

Calculate the average speed of the lorry in km/hr.

Answer _____ km/hr [3]

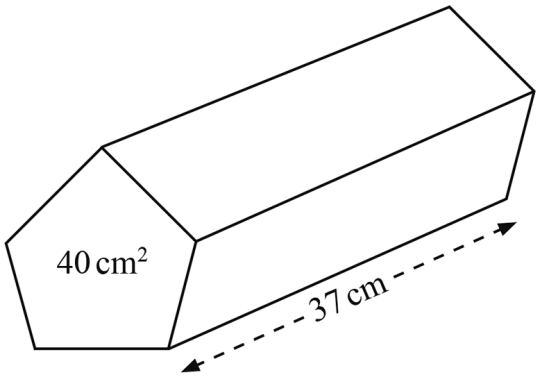
Q2 A train journey from Belfast to Dublin takes 2 hours and 15 minutes.

The distance travelled by the train is 144 kilometres.

Work out the average speed of the train in kilometres per hour.

Answer _____ km/h [2]

Q3 A solid pentagonal prism has mass 5300 g. The cross-sectional area is 40 cm^2 and the length is 37 cm.



Calculate the density of the prism in g/cm^3.

Give your answer to an appropriate degree of accuracy.

Answer _____ g/cm^3 [4]

Q4 Jane completes a 5 km race in 24 minutes.

Calculate her average speed in km/hr.

Answer _____ km/hr [2]

Q5 The table shows part of a train timetable from Edinburgh to St Andrews.

The Express trains travel directly. The Standard trains stop at other stations.

	Express	**Standard**	**Express**	**Standard**	**Express**
Edinburgh	1318	1343	1424	1441	1520
Haymarket		1406		1504	
Kirkcaldy		1418		1516	
Ladybank		1423		1521	
St Andrews	1403	1439	1509	1537	1605

(a) Alex arrives at Edinburgh Airport at 1306

It takes him 26 minutes to collect his luggage.

By taxi, he arrives at Edinburgh Train Station 18 minutes later.

How long will he have to wait at the station for the next train to St Andrews?

Answer _____ minutes [3]

(b) The distance between Edinburgh and St Andrews is 54 miles.

Calculate the average speed at which the Express train travels between Edinburgh and St Andrews.

Answer _____ miles/hr [3]

Q6

(b) A horse runs 2000 metres at an average speed of 14.5 m/s.

How long does this take?

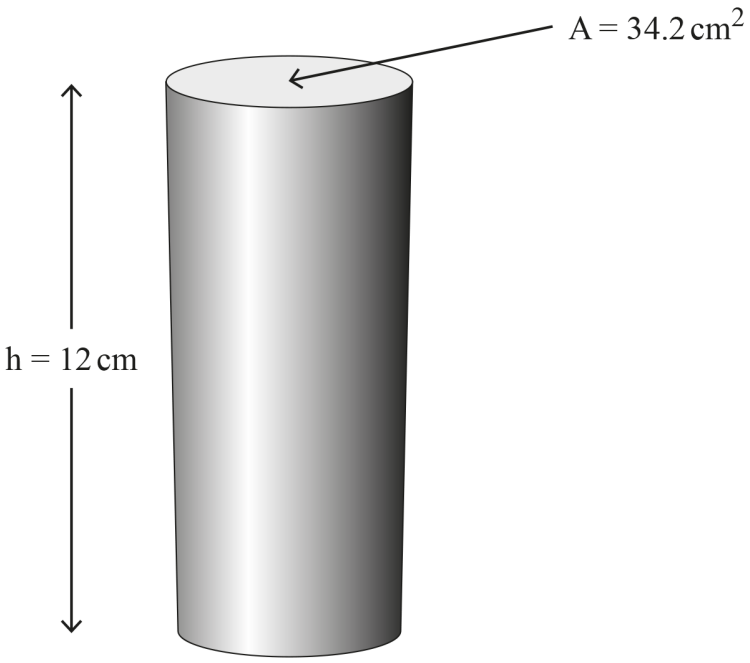
Give your answer in minutes and seconds, to the nearest second.

Answer _____ minutes _____ seconds [3]

Q7 A solid cylinder has a height of 12 cm and a circular cross-sectional area of $34.2\,cm^2$

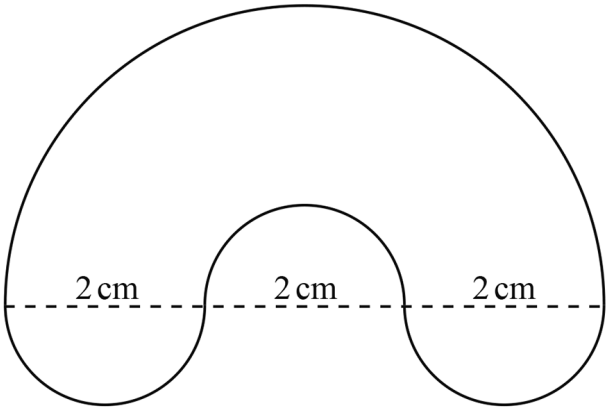
The density is $0.83\,g/cm^3$

Find the mass of the cylinder.



Answer _____ g [3]

Q8 A shape with four semicircular edges is shown.

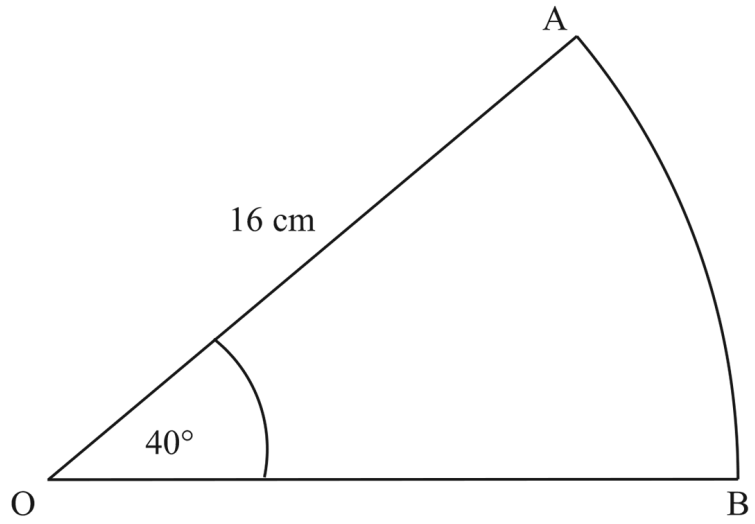


Calculate the perimeter of the shape.

Answer _____ cm [4]

Q9 AOB is a sector of a circle, radius 16 cm.

Angle AOB = 40°



Work out the perimeter of the sector AOB.

Answer _____ cm [3]

Q10

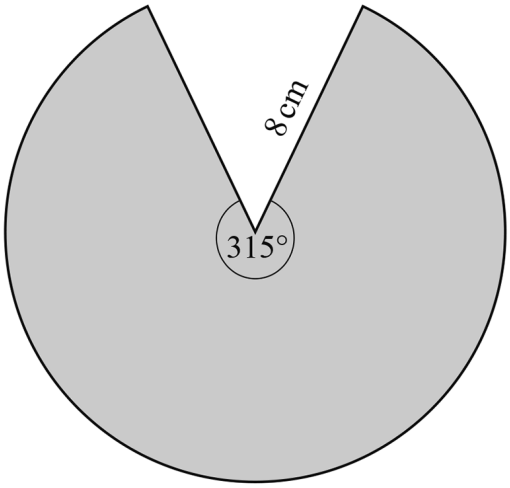
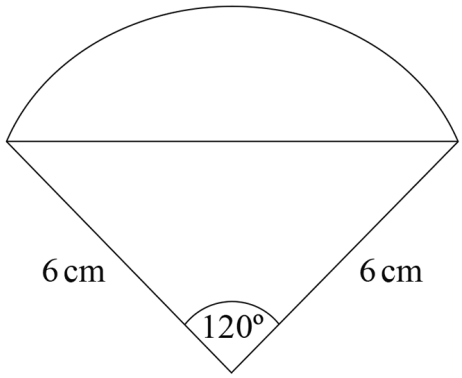


diagram not drawn accurately

Calculate the area of the shaded sector.

Answer _____ cm^2 [3]

Q11 The diagram shown is a sector of a circle of radius 6 cm with an angle of 120º



Calculate the area of the segment.

Answer _____ cm^2 [4]

Q12

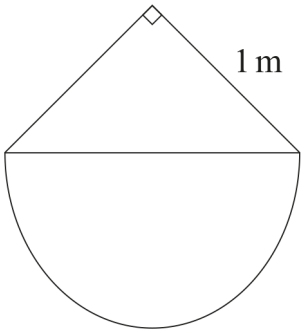


diagram not
drawn accurately

The composite shape consists of a right-angled isosceles triangle and a semicircle.

(a) Show that the area of the composite shape is approximately $1.285\,m^2$

[4]

(b) Find the force applied to the area of the composite shape when the pressure is $5\ N/m^2$

Answer _____ N [2]

Q13

Sue is training to compete in a 10 km walk.

A diagram of her local athletics track is shown below.

The track consists of a rectangle and two semicircles.

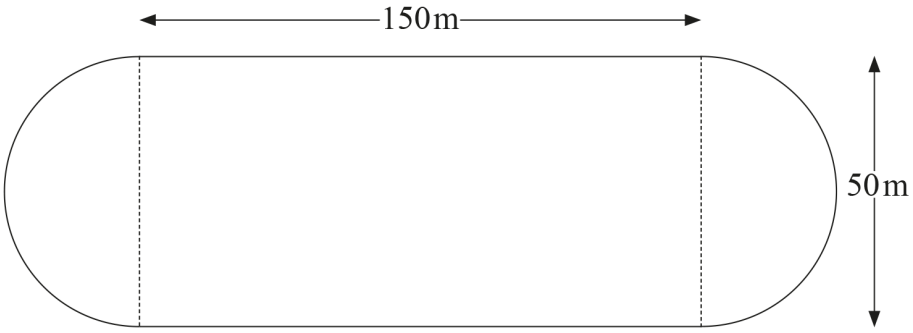


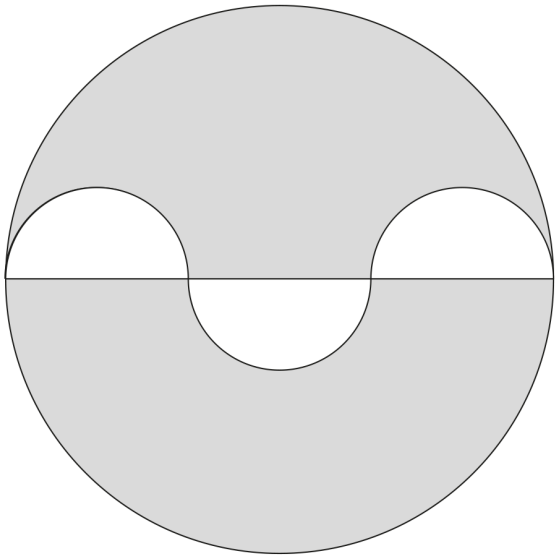
diagram not drawn accurately

How many complete laps are needed to ensure she walks 10 km?

You must show all your working.

Answer _____ [4]

Q14



A large circle has three semicircles of equal diameters placed across its diameter as shown.

The radius of each of the small semicircles is 2 cm.

Work out the area shaded.

Answer _____ cm^2 [5]

Q15 Find the area of the shaded sector of this circle, centre O and radius 3 cm.

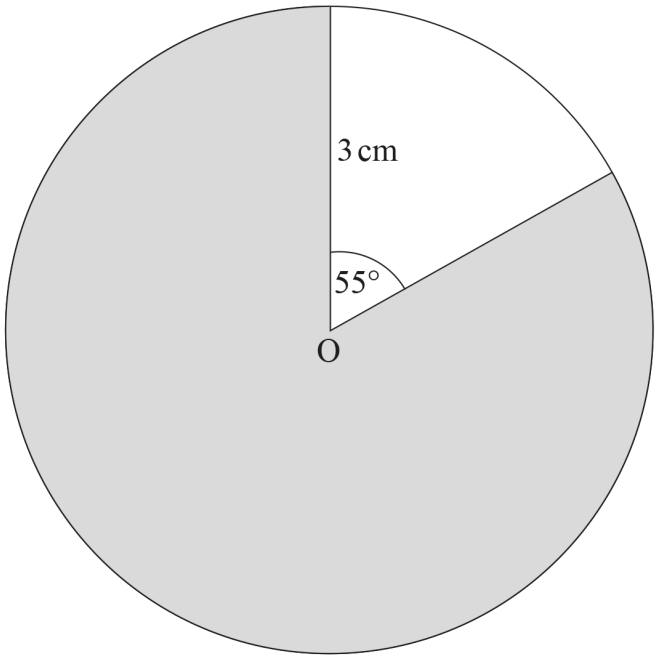
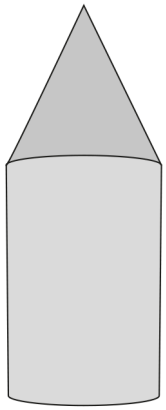


diagram
not drawn
accurately

Answer _____ cm^2 [2]

Q16 The roof on a tower is in the shape of a cone with a diameter of 7.5 m and a height of 5 m.

Calculate the volume of this cone.



Answer _____ m^3 [2]

Q17 A cylinder has a base radius of 8 cm and a height of 36 cm.

The curved surface area of this cylinder is the same as the surface area of a sphere.

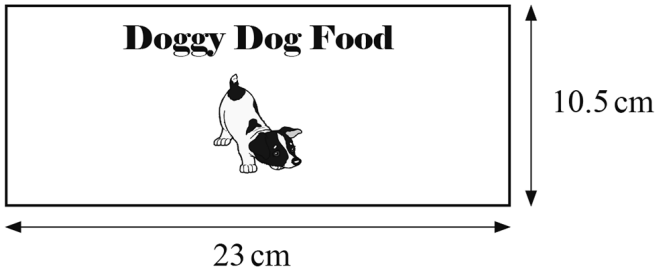
What is the radius of the sphere?

Answer _____ cm [4]

Q18 The picture shows the dimensions of a label taken from a cylindrical tin of dog food.

The label covers all the curved surface of the tin with no overlap.

Calculate the volume of the tin.



Answer _____ cm^3 [4]

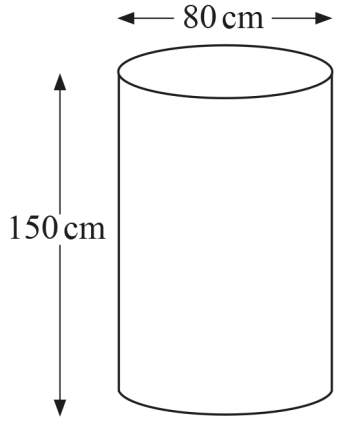
Q19 A metal cube made of lead is melted down to make 100 identical spherical weights with a radius of 2 cm.

Calculate the smallest **integer** side length for the cube.

Answer _____ cm [4]

Q20

A cylindrical tank has a diameter of 80 cm and a height of 150 cm as shown.

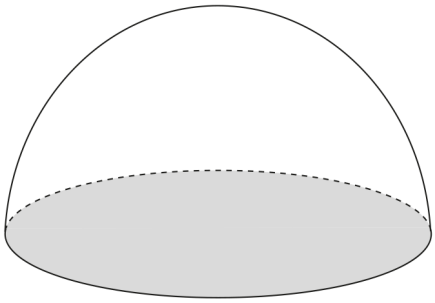


Calculate the volume of water the tank can hold when full.

Give your answer correct to the nearest litre.

Answer _____ litres [4]

Q21 The **solid** hemisphere has a diameter of 12 cm.



Mary says the total surface area is 226 cm^2 to the nearest cm^2

Martha says the total surface area is 339 cm^2 to the nearest cm^2

Explain with reasoning who is correct.

Answer _____ is correct [4]

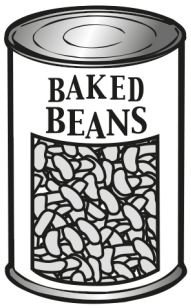
Q22 Calculate the surface area of a sphere with diameter 12 cm.

Answer _____ cm^2 [2]

Q23 A cylindrical can of beans has a radius of 3.4 cm and a height of 12 cm.

The entire curved surface area is covered by a label.

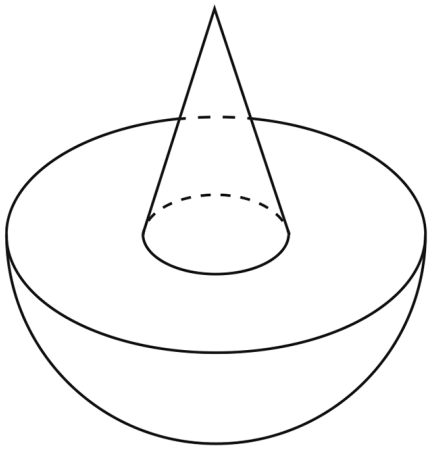
The label has a 1 cm overlap to allow for sticking.



Calculate the area of the label.

Answer _____ cm^2 [3]

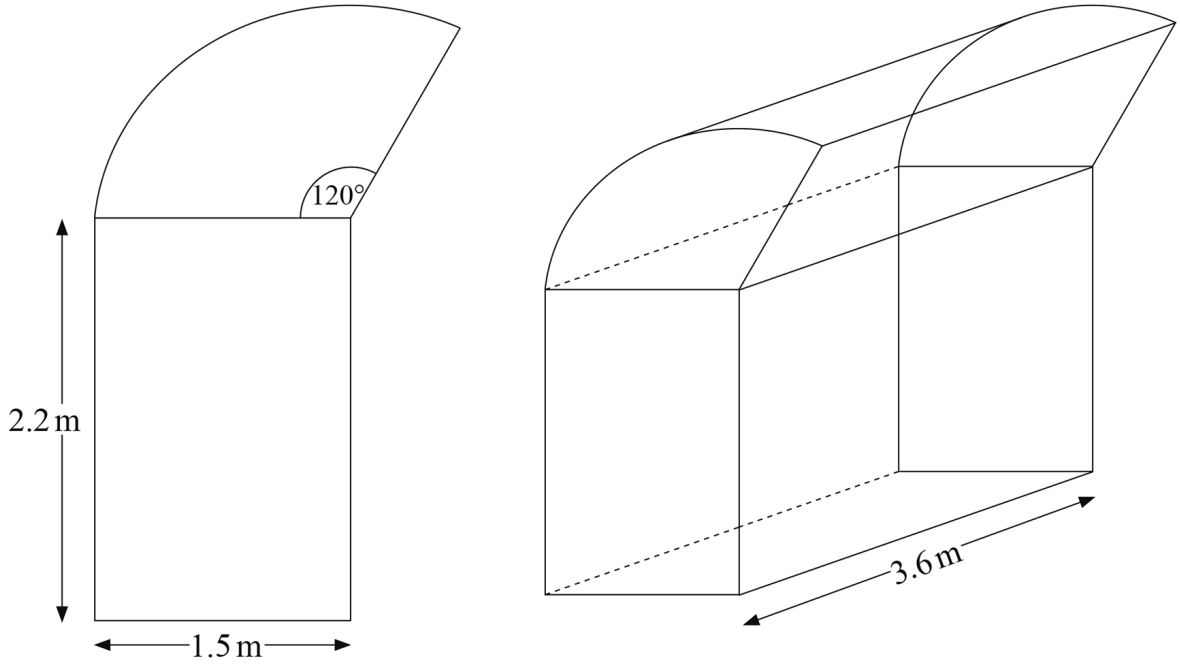
Q24 A solid wooden spinner is made up of a cone attached to a hemisphere as shown.



The hemisphere has a diameter of 16cm.
The base radius of the cone is 3cm and the vertical height of the cone is 10cm.
The surface of the spinner is to be painted. Calculate the surface area of the spinner.
Give your answer correct to 3 significant figures.

Answer _____ cm^2 [6]

Q25



A B

Diagram A above represents the cross section of a solid sculpture (B).

The lower section is a rectangle measuring 1.5 metres by 2.2 metres.

The upper section is a sector of a circle containing an angle of 120°

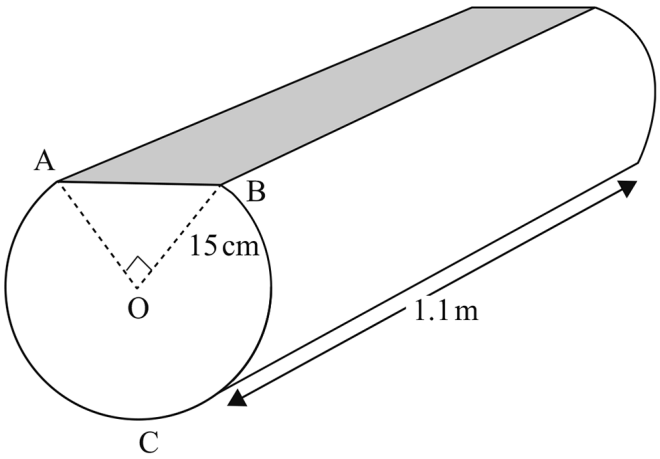
The sculpture is 3.6 metres long.

Work out the volume of the sculpture.

Answer _____ m^3 [4]

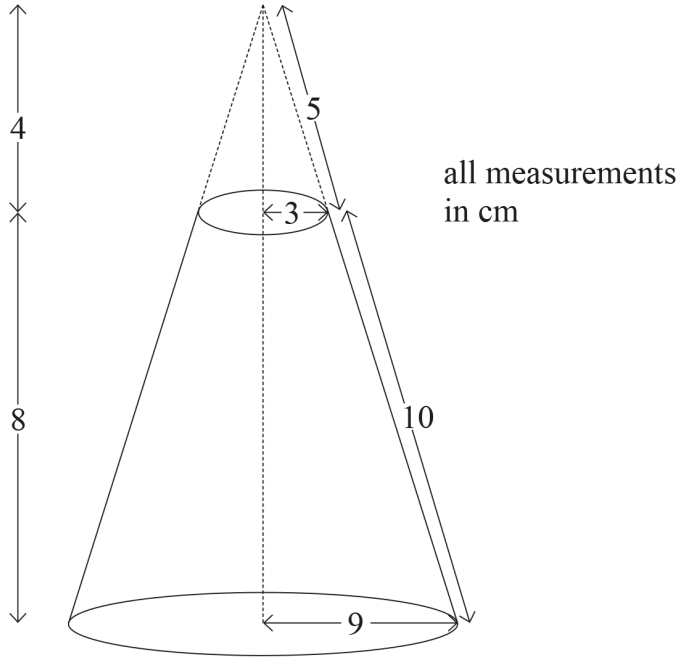
Q26 A cylindrical wooden log of length 1.1 m has been sliced along the upper end to leave a flat top with a uniform cross section as shown.

ACB is the arc of a circle of radius 15 cm. Angle AOB = 90°
Calculate the remaining volume.



Answer _____ m^3 [5]

Q27 The diagram represents the frustum of a solid cone formed when a small cone is removed from the top of a large cone.



Work out the total surface area of the frustum.

You must show all your working.

Answer: _____ cm^2 [4]

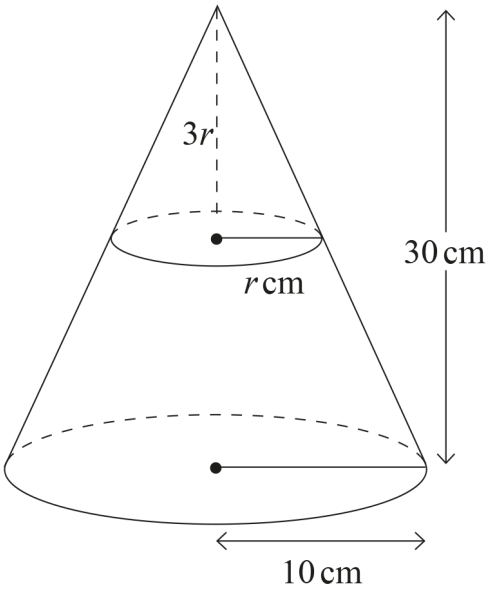
Q28

A cone of radius r cm and height $3r$ cm is removed from a cone of radius 10 cm and height 30 cm to give a frustum.

The volume of the frustum is 2855 cm^3

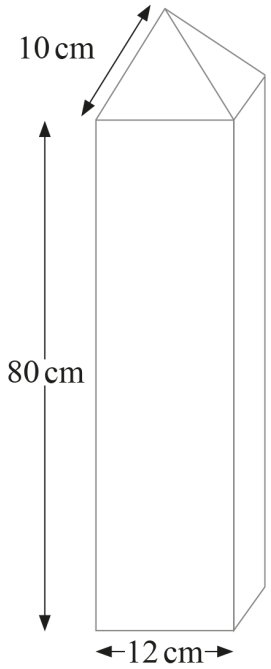
Calculate the value of r.

Show all your working.



Answer _____ [6]

Q29 The diagram shows a wooden post consisting of a cuboid with a square-based pyramid on top. Each slanted edge is 10 cm.



Calculate the total surface area of the wooden post.

Answer_____ cm^2 [5]

Q30

The diagram shows a sector AOB of a circle, with radius 13 cm and centre O.

The point C lies on OB and angle ACO is 90°

OC = 5 cm.

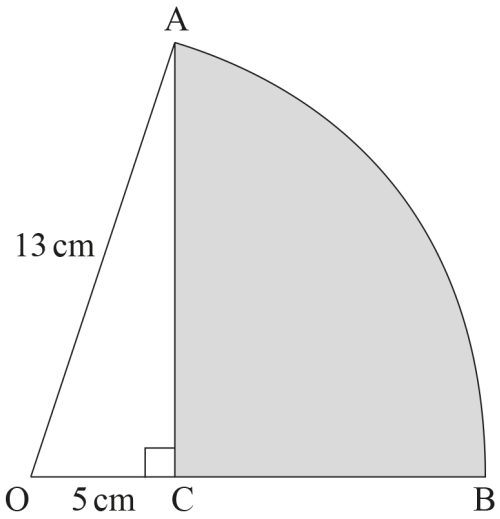


diagram not drawn accurately

Find the area of the shaded section ABC.

Answer _____ cm^2 [8]

Q31

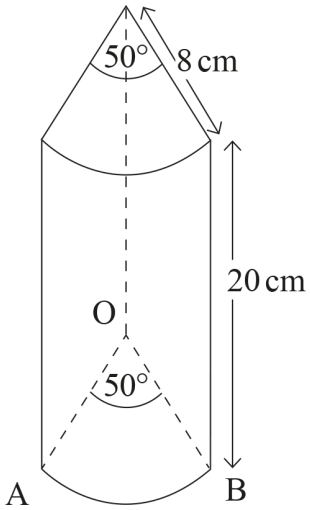


diagram
not drawn
accurately

A solid prism has base OAB, the sector of a circle, and height 20 cm.

Radius OA = 8 cm and angle AOB = 50°.

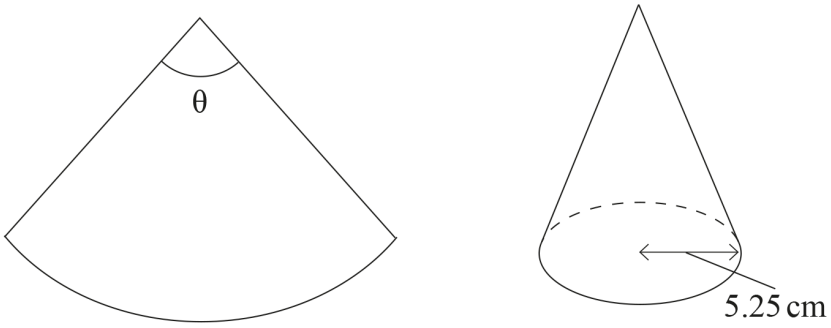
Calculate the total surface area of the prism.

Answer _____ cm^2 [7]

Q32

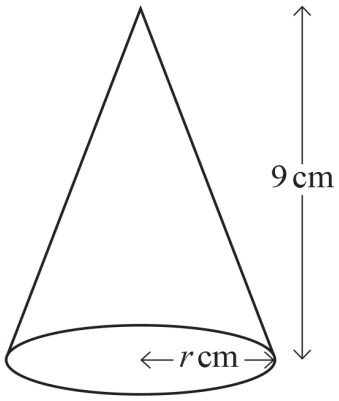
A cone with a base radius of 5.25 cm and a volume of 497 cm^3 is made by folding a sector as shown.

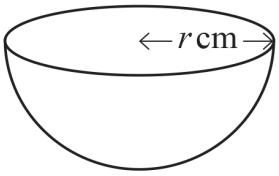
Calculate (to the nearest degree) the angle θ at the apex of the sector needed to form the cone.



Answer _____ ° [7]

Q33





diagrams
not drawn
accurately

A solid cone has a radius of r cm and a perpendicular height of 9 cm.

A solid hemisphere also has a radius of r cm.

The surface areas of both shapes are equal.

Calculate the value of the radius r.

Answer $r =$ _____ [6]

Q34 A solid metal cylinder has a base radius of $3x$ and a height of $32x$.

The cylinder is melted down and made into a sphere of radius r.

All lengths are in cm.

Find an expression for r in terms of x.

Answer r = _____ [4]

1. 150 (mins) = $2\frac{1}{2}$ (hours) A1

$240 \div 2\frac{1}{2} = 240 \times \frac{2}{5}$ M1

= 96 km/hr A1

alternatively

240 km = 150 mins (÷5) MA1
48 km = 30 mins (× 2) MA1
96 km = 60 mins (96 km/hr) A1

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2. 144 ÷ 2.25 (o.e.) M1
64 A1

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3. 40 × 37 = 1480 MA1
5300 ÷ 1480 = 3.58(108) M1 A1
3.6 or 4 MA1

4.

speed = $\frac{5}{\left(\frac{24}{60}\right)}$ or $5 \div 0.4$ MA1

$= 12.5$ km/hr A1

alternative solution

5 km = 24 minutes ($\div 4$)
1.25 km = 6 mins ($\times 10$) MA1
12.5 km = 60 mins = 1 hour A1

5.

(a) 1306 + 26 minutes + 18 minutes M1

$= 1350$ A1

Has to wait 34 minutes A1

(b) 1424 – 1509 = 45 mins ($\frac{3}{4}$ hr) MA1

Speed = $\frac{54}{\frac{3}{4}}$ **or** 54 miles in 45 minutes

18 miles in 15 mins [or 1.2 miles in 1 min] MA1

$= 72$ A1

6.

(b) $2000 \div 14.5$ MA1

137.9310..... A1

2 minutes 18 seconds A1

7.

$34.2 \times 12 = 410.4$	MA1
0.83×410.4	MA1
$= 340.632$	A1

8.

Circumference of large circle = $\pi d = 18.84955592$	MA1
Outer edge of large semicircle = $(\pi \times 6)/2 = 9.424777961$	MA1
Outer edge of 1 small semicircle = $(\pi \times 2)/2 = 3.141592654$	MA1
Total = $9.424777961 + 3 \times 3.141592654 = 18.8$ (4955592)	MA1

9.

$2\pi r = 100.5309649$	MA1
$\frac{1}{9}$ of $(100.5309649) = 11.17$	MA1
43.17	A1

10.

$\frac{315}{360} \times \pi \times 8^2$	MA2
$= 175.9\ \text{cm}^2$	A1

11.

Area of sector $= \frac{120}{360} \times \pi \times 6^2 = 12\pi = 37.699$	MA1 A1
Area of triangle $= \frac{1}{2} \times 6 \times 6 \times \sin 120 = 15.588$	MA1
Area of segment $= 37.699 - 15.588 = 22.111$	A1

12.

(a)	$d^2 = 1^2 + 1^2$	M1
	$d = 1.4142 \ldots$	A1
	Area of half square $= 0.5$	MA1
	Area of semi-circle ? $= \frac{1}{2} \times \pi \times 0.707^2$ $= 0.785$	MA1
	Total area $= 1.285$	
(b)	$F = 5 \times 1.285$ $= 6.42(58 \ldots)$	M1 A1

13.

$\pi \times 50 = 157(.0796...)$	MA1
$157(.0796...) + 300 = 457(.0796...)$	MA1
$10000 \div 457(.0796...)$	M1
$21.8(7802...)$ so she needs to walk 22 laps	A1

14.

Area of large circle $= \pi \times 6^2 = 113.0973355$	M1 A1
Area of each semicircle $= \frac{1}{2} \times \pi \times 2^2 = 6.283185307$	MA1
Shaded area $= 113.0973355 - 3 \times 6.283185307$ (must use 3 semicircles)	M1
$= 94.24777961$	A1

15.

$A = \frac{305}{360} \times \pi \times 3^2$	MA1
$= 23.95\ (464\,398)$	A1
accept any correct rounding	

16.

$V = \frac{1}{3} \pi \times 3.75^2 \times 5$	MA1
$= 73.6\,m^3$	A1

17.

$2\pi \times 8 \times 36$	M1
576π	A1
$4\pi r^2 = 576\pi$	M1
$r^2 = 144,\ \therefore r = 12$	A1

18.

Radius $= 23 \div 2\pi = 3.66\,(056369\ldots)$	M1 A1
Volume $= \pi \times (3.66)^2 \times 10.5$	MA1
$= 442.0\,(130657\ldots)$	A1

19.

Volume of lead needed $= \frac{4}{3}\pi \times 2^3 \times 100$	M1
$= 3351.032164\,\text{cm}^3$	A1
$\sqrt[3]{3351.03\,(2164)} = 14.96$	MA1
Side length $= 15\,\text{cm}$	A1

20.

$\pi \times 40^2 \times 150$	MA2
$= 753982.2369$	A1
$= 754$ litres	MA1

21.

SA of sphere $= 4 \times \pi \times 6^2 = 452$	MA1
SA hemisphere $= 226$	MA1
base $= \pi \times 6^2 = 113$	MA1
total $= 339$	
Martha is correct	MA1

22.

$4 \times \pi \times 6^2$	MA1
$= 452.38(93421)$	A1

23.

$2 \times \pi \times 3.4 = 21.3628(3004)$	MA1
$21.3628 + 1 = 22.3628$	MA1
$22.3628 \times 12 = 268.35(39605)$	MA1
alternative solution	
$2 \times \pi \times 3.4 \times 12 = 256.3539(605)$	MA2
$256.3539(605) + 1 \times 12 = 268.35(39605)$	MA1

24.

Curved SA of hemisphere $= 2 \times \pi \times 8^2 = 402.1238597$ or 128π	MA1
Top of hemisphere $= \pi \times 8^2 - \pi \times 3^2$	M1
$= 172.7875959$	A1
Slant height of cone $= \sqrt{(3^2 + 10^2)} = 10.44030651$	MA1
csa cone $= \pi \times 3 \times 10.44030651 = 98.39757069$	MA1
Total $= 673\text{cm}^2$	A1

25.

Area of cross section $= (2.2 \times 1.5) + \frac{1}{3}\pi \times 1.5^2$	M1
5.656	A1
Volume $= 5.656 \times 3.6$	MA1
20.3623 or (20.3616)	A1

Alternative solution

Volume of cuboid $= 1.5 \times 2.2 \times 3.6 = 11.88$	MA1
$\frac{1}{3}$ volume of cylinder $= \frac{1}{3}\pi \times 1.5^2 \times 3.6$	M1
8.4823	A1
Total volume = 20.3623	A1

26.

Major sector area $= \frac{270}{360} \times \pi \times 15^2$	M1
530.14	A1
Triangle area $= \frac{1}{2} \times 15 \times 15$ (112.5)	
Cross-sectional area = 530.14 + 112.5 (642.64)	MA1
Volume of wood = 642.64 × 110 (70690.8)	MA1
$= 0.0707\ m^3$	A1

27.

CSA of frustum	$= \pi \times 9 \times 15$	$-$	$\pi \times 3 \times 5$	
		$=$	376.9911184	M1 A1
Area of top and base	$= 9\pi + 81\pi$	$=$	282.7433388	MA1
Total area		$=$	659.73(44578)	A1

28.

Volume of large cone $= \frac{1}{3} \times \pi \times 10^2 \times 30 = 3141.592654$ MA1

Volume of small cone $= \frac{1}{3} \times \pi \times r^2 \times 3r = 3.141592654r^3$ MA1

$3141.592654 - 3.141592654r^3 = 2855$ M1 A1

$3.141592654r^3 = 286.5926536$

$r^3 = 91.22527495$ MA1

$r = 4.5$ A1

(work must be shown but do not penalise early rounding;
correct answer with no work gains no marks)

29.

Height of triangles $h^2 = 10^2 - 6^2$ M1

$h = 8\,\text{cm}$ A1

Surface Area $= 4(12 \times 80) + (12 \times 12) + 4(\frac{1}{2} \times 12 \times 8)$ MA2

$= 4176\,\text{cm}^2$ MA1

30.

$\text{Cos AOC} = \frac{5}{13}$

AOC = 67.4°	M1 A1
Area of sector AOB = $\frac{67.4}{360} \times \pi \times 13^2 = 99.4$	M1 A1
$AC^2 = 13^2 - 5^2$	
AC = 12	M1 A1
Area of triangle OAC = $\frac{1}{2} \times 5 \times 12 = 30$	MA1
Shaded Area = $99.4 - 30 = 69.4\text{cm}^2$	MA1

31.

curved surface area = $\frac{50}{360} \times 2 \times \pi \times 8 \times 20 = 139.626....$	M1 A1
sides = $2 \times 8 \times 20 = 320$	MA1
$\pi \times 8^2$ (= 201.061...)	MA1
area sector AOB = $\frac{50}{360} \times \pi \times 8^2 = 27.925....$	M1 A1
total area = 2(27.925....) + 139.626.... + 320 = 515(.47687....)	MA1

32.

$\frac{1}{3} \times \pi \times 5.25^2 \times h = 497$ MA1

$h = 17.21904908$ A1

$l^2 = 17.21904908^2 + 5.25^2$ MA1

$l = 18.00161524$ A1

$\frac{\theta}{360} \times 2 \times \pi \times 18.00161524 = 2 \times \pi \times 5.25$ MA2

$\theta = 105$ A1

alternative solution

$\frac{1}{3} \times \pi \times 5.25^2 \times h = 497$ MA1

$h = 17.21904908$ A1

$l^2 = 17.21904908^2 + 5.25^2$ MA1

$l = 18.00161524$ A1

$\frac{\theta}{360} \times \pi \times 18.00161524^2 = \pi \times 5.25 \times 18.00161524$ MA2

$\theta = 105$ A1

33.

$l^2 = 9^2 + r^2$ MA1

$l = \sqrt{81 + r^2}$

S.A. of cone $= \pi r^2 + \pi rl$

$\pi r^2 + \pi r\sqrt{81 + r^2}$ MA1

S.A. of hemisphere $= \pi r^2 + \frac{1}{2}(4\pi r^2)$ MA1

$= 3\pi r^2$

$3\pi r^2 = \pi r^2 + \pi r\sqrt{81 + r^2}$ MA1

$2\pi r^2 = \pi r\sqrt{81 + r^2}$

$2r = \sqrt{81 + r^2}$ MA1

$4r^2 = 81 + r^2$

$r = 3\sqrt{3}$ (5.1961524230) A1

34.

$\frac{4}{3} \times \pi \times r^3 = \pi \times (3x)^2 \times 32x$ M1

$\frac{4}{3}\pi r^3 = 288\pi x^3$ MA1

$r^3 = 216x^3$ MA1

$r = 6x$ A1
