St. Patrick's High School, Keady

Mathematics Department

GCSE Mathematics Practice Booklet

M8

Topic 4 –Number 2

Standard Form

Surds

Recurring Decimals

Irrational Numbers

Growth and Decay

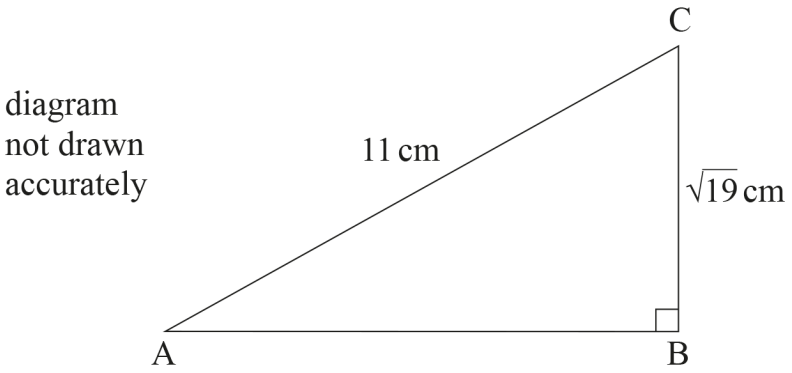
Section A – **Non-Calculator Questions** / Mark Scheme Pages 1-49

Section B – **Calculator Questions** / Mark Scheme Pages 50-56

Questions taken from CCEA Past Papers

Keady Maths

Q1



ABC is a right-angled triangle.

Work out the length of AB, giving your answer as a surd.

Answer _____ cm [3]

Q2

A tent in the shape of a cone has a perpendicular height of 7 m and a volume of $220\,m^3$

Using $\pi = \frac{22}{7}$, work out the base radius of this tent.

Write your answer in surd form.

Answer _____ [3]

Q3

$m = 4.5 \times 10^{7}$ $n = 5 \times 10^{-3}$

Work out the value of $\frac{m}{n}$, giving your answer in standard form.

Answer _____ [2]

Q4

In standard form

$$(2 \times 10^{x}) \times (3 \times 10^{y}) = 6 \times 10^{11}$$

$$(3 \times 10^{y}) \div (2 \times 10^{x}) = 1.5 \times 10^{3}$$

Find the values of x and y.

Show your working.

Answer x = _____ y = _____ [4]

Q5

$A = 2.5 \times 10^{8}$

$B = 8 \times 10^{-2}$

$C = 5 \times 10^{4}$

Work out the value of $\frac{AB}{C}$

Give your answer in standard form.

Answer _____ [3]

Q6 $1\,000\,000\,cm^3$ of gas has mass of 1.5×10^4 g.

Giving your answer in standard form, calculate the mass of one cubic centimetre of the gas.

Answer _____ g [2]

Q7 A one gram bag of seed contains half a million seeds.

If each seed weighs the same, calculate the weight, in grams, of one seed.

Give your answer in standard form.

Answer _____ g [3]

Q8

The mass of Particle P has been recorded as 0.000 000176 g.
The mass of Particle Q has been recorded as 14.9×10^{-7} g.
Which particle has the bigger mass and by how much?
Give your answer in standard form.

Answer Particle _____ by _____ g [3]

Q9

(a) Work out $6.543 \times 10^3 + 2.1 \times 10^{-1}$

Give your answer in standard form.

Answer _____ [2]

(b) Given that $(3.6 \times 10^3) + (1.7 \times 10^x) = (y \times 10^5)$ where all three numbers in brackets are in standard form, find the values of x and y.

Answer $x =$_____, $y =$_____ [2]

Q10 **(a)** Work out and simplify

$$(\sqrt{3} + \sqrt{27})^2$$

Answer _____ [2]

(b) $(6 - \sqrt{5})(3 + 2\sqrt{5}) = a + c\sqrt{5}$

Find the values of a and c.

Answer a = _____ , c = _____ [3]

Q11 Simplify $4\sqrt{3} + 2\sqrt{75} + \sqrt{27}$

Answer _____ [2]

Q12 By expanding each bracket and simplifying, explain why

$(3 + \sqrt{5})^2 - (2 + \sqrt{2})^2 > 0$

[3]

Q13

$$\frac{x\sqrt{2}}{5-\sqrt{5}} = 5 + \sqrt{5}$$

Work out the value of x.
Give your answer in the form $a\sqrt{b}$ where a and b are integers.

Answer _____ [4]

Q14 Rationalise the denominator of $\frac{54}{\sqrt{3}}$ and simplify your answer.

You must show your working.

Answer _____ [2]

Q15 Expand $(2 + \sqrt{7})(3 - \sqrt{7})$

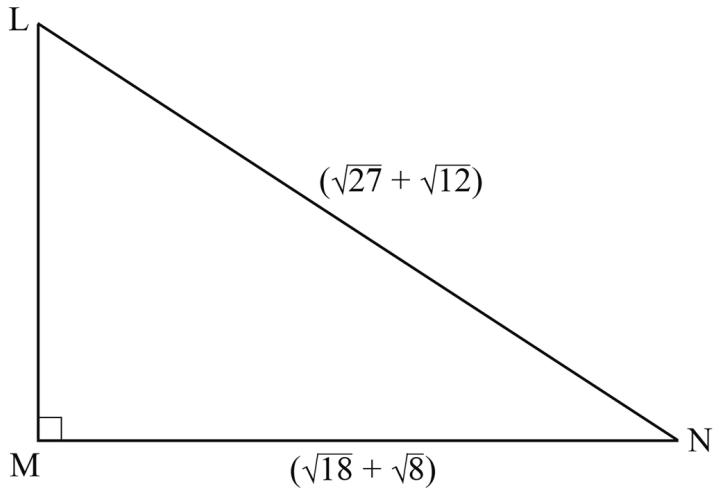
Give your answer in the form $a + b\sqrt{7}$ where a and b are both integers.

Answer _____ [2]

Q16 Write $\frac{\sqrt{125}-\sqrt{45}}{\sqrt{125}+\sqrt{45}}$ in its simplest form.

Answer _____ [3]

Q17



LMN is a right-angled triangle with angle M = 90°

LN = $(\sqrt{27} + \sqrt{12})$ cm

MN = $(\sqrt{18} + \sqrt{8})$ cm

Show that LM = 5 cm.

[4]

Q18

(a) Rationalise the denominator of $\frac{28}{\sqrt{7}}$

Answer _____ [2]

(b) Show that $(\sqrt{45} - \sqrt{5})^2 = 20$

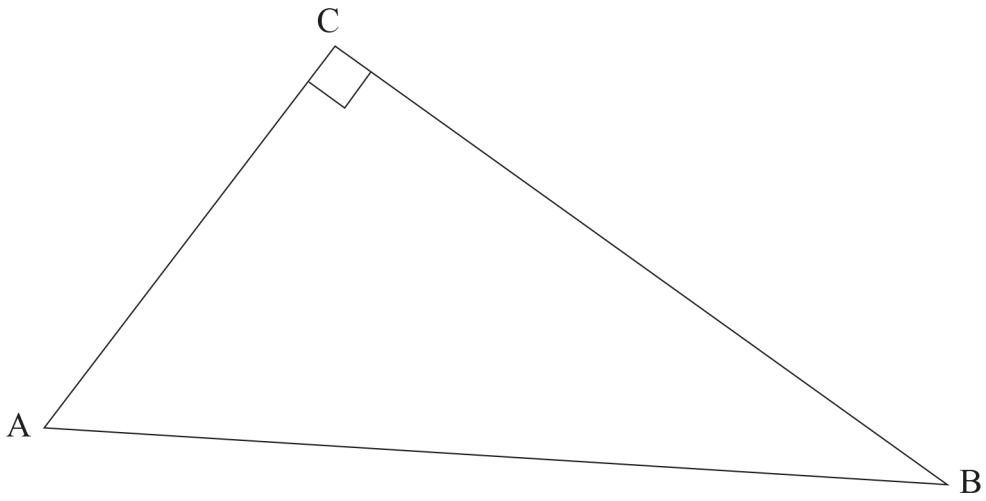
[2]

Q19

Circle the irrational numbers in the list

$\frac{\pi^2}{4}$ $\frac{\sqrt{27}}{\sqrt{2}}$ $\frac{\sqrt{27}}{\sqrt{3}}$ $\frac{\sqrt{27}}{\sqrt{4}}$ $\sqrt[3]{27}$ [3]

Q20



BC $= 3\sqrt{5} - 1$ and AC $= 3 + \sqrt{5}$

Find AB.

Give your answer in the form $p\sqrt{q}$

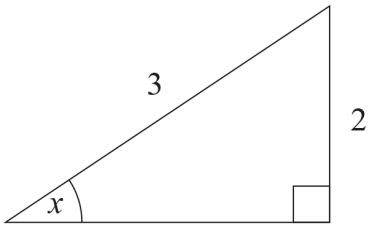
[5]

Q21

Work out $\frac{4}{\sqrt{3}} + \frac{\sqrt{3}(\sqrt{27} - 8)}{6}$

Answer _____ [4]

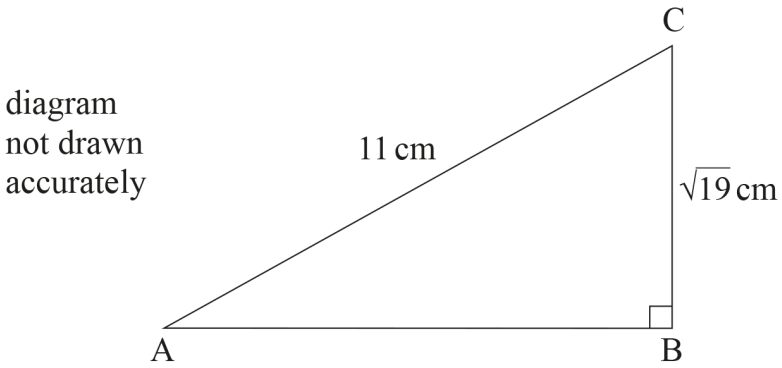
Q22



Show that $\cos x + \tan x = \frac{11\sqrt{5}}{15}$

[5]

Q23



ABC is a right-angled triangle.

Work out the length of AB, giving your answer as a surd.

Answer _____ cm [3]

Q24 A cuboid has sides of length

$$3 + 2\sqrt{3}, \qquad 3 + \sqrt{3}, \qquad 9 - \sqrt{3}$$

Find the length of the space diagonal, giving your answer in the form $a\sqrt{b}$

Answer _____ [5]

Q25

$\frac{3}{4\sqrt{5}}$ can be expressed in the form $a\sqrt{5}$

Find the value of a

Answer a = _____ [2]

Q26

A tent in the shape of a cone has a perpendicular height of 7 m and a volume of $220\,m^3$

Using $\pi = \frac{22}{7}$, work out the base radius of this tent.

Write your answer in surd form.

Answer _____ [3]

Q27

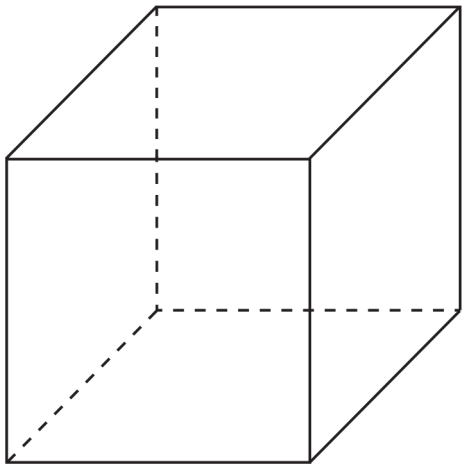
$$(2 + a\sqrt{3})^2 = b + 20\sqrt{3}$$

Work out the values of the integers a and b.

Show all your working clearly.

Answer a = _____, b = _____ [4]

Q28



The length of the space diagonal of a cube is 9 cm.

Find the length of a side of the cube, giving your answer in the form $a\sqrt{b}$

Answer _____ cm [3]

Q29 **(a)** Write 0.000108 in standard form.

Answer _____ [1]

(b) Calculate $5.6 \times 10^5 \div 1.4 \times 10^2$

Answer _____ [1]

(c) Change the recurring decimal 0.727272 ..... into a fraction in its simplest form.

Answer _____ [2]

Q30 Write the recurring decimal $0.2\dot{1}\dot{7}$ as a fraction.

Answer _____ [2]

Q31 Write $0.3\dot{1}\dot{8}$ as a fraction.

Answer _____ [2]

Q32 Express the recurring decimal $0.3\dot{7}\dot{2}$ as a fraction in its simplest form.

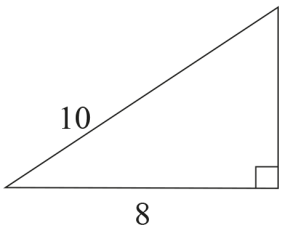
Answer _____ [3]

Q33 Change the recurring decimal 0.561561 … into a fraction in its simplest form.

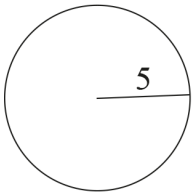
Answer _____ [2]

Q34

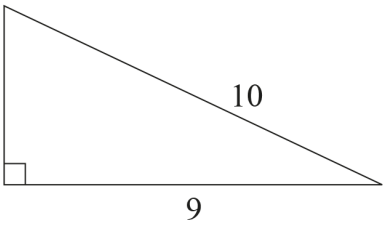
For each shape, decide whether the area is a rational or an irrational number.
Give a reason for each answer.



Shape A



Shape B



Shape C

Area of Shape A is _____ because _____

_____ [1]

Area of Shape B is _____ because _____

_____ [1]

Area of Shape C is _____ because _____

_____ [1]

Q35

In each case decide if x, y or z is rational or irrational, giving a reason for your answer.

(a) x is the radius of a circle of circumference 12π

Answer x is _____ because _____ [1]

(b) y is the radius of a circle of area $9\pi^2$

Answer y is _____ because _____ [1]

(c) z is the radius of a circle of area 8π

Answer z is _____ because _____ [1]

Q36 Change 0.0858585… into a fraction.

Answer _____ [2]

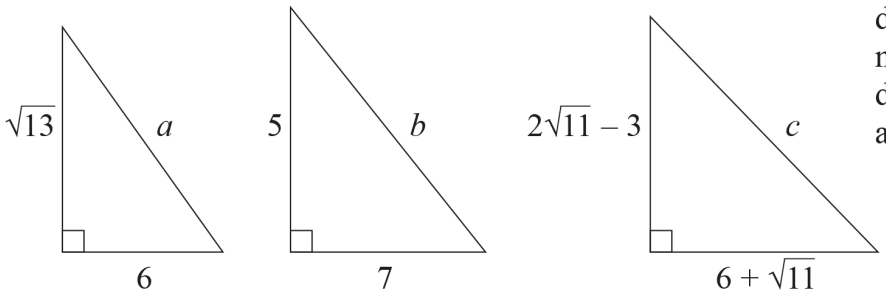
Q37 A circle of radius r cm has circumference C cm and area A cm^2

Circle whether each value is rational or irrational in the table.

r	C	A
$\frac{3}{\pi}$	rational / irrational	rational / irrational
$\sqrt{\frac{3}{\pi}}$	rational / irrational	rational / irrational
$3\sqrt{\pi}$	rational / irrational	rational / irrational

[4]

Q38



diagrams not drawn accurately

For each of the right-angled triangles above, state whether the length of the hypotenuse is rational or irrational, giving reasons for your answers.

Answer *a* is _____ because _____

b is _____ because _____

c is _____ because _____

[6]

Q39 Change 0.08181818181… into a fraction.

Answer _____ [2]

Q40

$$A = (7 - \sqrt{5})(7 + \sqrt{5}) \qquad B = \left[(7 - \sqrt{5}) - (7 + \sqrt{5})\right]^2 \qquad C = \frac{14\sqrt{10}}{7\sqrt{2}}$$

Prove that only one of A, B, C is irrational.

[5]

Q41

Tom bought shares costing £4 000

The value, V, of the shares depreciated by 0.05% each year.

Circle the formula which gives the value, V, of the shares after two years.

$V = (4000 - 0.05)^2$ $V = 4000\ (1.05)^2$

$V = 4000\ (0.9995)^2$ $V = 4000\ (0.95)^2$ [1]

1.

$AB^2 = 11^2 - (\sqrt{19})^2$ M1

$AB^2 = 102$ A1

$AB = \sqrt{102}$ A1

2.

$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 7 = 220$ MA1

$r^2 = 30$ MA1

$r = \sqrt{30}$ A1

3.

$\frac{4.5 \times 10^7}{5 \times 10^{-3}} = 0.9 \times 10^{10}$ MA1

9×10^9 A1

4.

$x + y = 11$ MA1

$y - x = 3$ or $x = y - 3$ or $y = x + 3$ MA1

$x = 4$ $y = 7$ A1 A1

5.

$\frac{20 \times 10^6}{5 \times 10^4}$ A1A1

$= 4 \times 10^2$ A1

6.

$1.5 \times 10^4 \div 10^6$ MA1

1.5×10^{-2} MA1

7.

$\frac{1}{500000} = \frac{1}{5 \times 10^5}$ MA1

0.2×10^{-5} MA1

2×10^{-6} A1

8.

$14.9 \times 10^{-7} = 0.000\ 00149$ A1

$0.000\ 00149 - 0.000\ 000176 = 0.000\ 001314$ MA1

Q by 1.314×10^{-6} A1

or

$0.000\ 000176 = 1.76 \times 10^{-7}$ A1

$14.9 \times 10^{-7} - 1.76 \times 10^{-7} = 13.14 \times 10^{-7}$ MA1

Q by 1.314×10^{-6} A1

9.

(a) $6543 + 0.21$ or 6543.21 MA1

6.54321×10^{3} A1

(b) $x = 5$ A1

$y = 1.736$ A1

10.

(a) $3 + 27 + 2\sqrt{81}$ or $(\sqrt{3} + 3\sqrt{3})^2 = (4\sqrt{3})^2$ MA1

48 A1

(b) $18 - 3\sqrt{5} + 12\sqrt{5} - 10$ MA1

$a = 8$ A1

$c = 9$ A1

11.

$4\sqrt{3} + 10\sqrt{3} + 3\sqrt{3}$	MA1
$17\sqrt{3}$	A1

12.

$9 + 6\sqrt{5} + 5 - (4 + 4\sqrt{2} + 2)$	C1
$8 + 6\sqrt{5} - 4\sqrt{2}$	C1
$\sqrt{2} < 2$, so $4\sqrt{2} < 8$, so $8 + 6\sqrt{5} - 4\sqrt{2} > 0$	C1

13.

Multiply both sides by $5 - \sqrt{5}$	M1
$x\sqrt{2} = 20$	A1
$x = \frac{20}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$	MA1
$x = \frac{20\sqrt{2}}{2} = 10\sqrt{2}$	A1

14.

$\frac{54}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ **or** $\frac{54\sqrt{3}}{3}$	M1
$18\sqrt{3}$	A1

15.

$6 - 2\sqrt{7} + 3\sqrt{7} - 7$ MA1

$-1 + \sqrt{7}$ A1

16.

$\frac{5\sqrt{5} - 3\sqrt{5}}{5\sqrt{5} + 3\sqrt{5}}$ MA1

$= \frac{2\sqrt{5}}{8\sqrt{5}}$ A1

$= \frac{1}{4}$ A1

17.

$LN = 5\sqrt{3}$ C1

$MN = 5\sqrt{2}$ C1

$LM^2 = (5\sqrt{3})^2 - (5\sqrt{2})^2 = 75 - 50 = 25$ C1

$LM = 5$ C1

or

$LN^2 = (\sqrt{27} + \sqrt{12})(\sqrt{27} + \sqrt{12})$

$= 27 + 2\sqrt{27}\sqrt{12} + 12$

$= 39 + 2 \times 3\sqrt{3} \times 2\sqrt{3}$

$= 39 + 36 = 75$ C1

$MN^2 = (\sqrt{18} + \sqrt{8})(\sqrt{18} + \sqrt{8})$

$= 18 + 8 + 2\sqrt{18} \times \sqrt{8}$

$= 26 + 2 \times 3\sqrt{2} \times 2\sqrt{2}$

$= 26 + 24 = 50$ C1

$LM^2 = 75 - 50 = 25$ C1

$LM = 5$ C1

18.

(a) $\frac{28}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$ A1

$4\sqrt{7}$ A1

(b) $(\sqrt{45} - \sqrt{5})^2 = 45 - 2\sqrt{225} + 5$ M1

$50 - 2 \times 15 =$

$50 - 30 = 20$ A1

or Alternative

$(\sqrt{45} - \sqrt{5})^2 = (3\sqrt{5} - \sqrt{5})^2 = (2\sqrt{5})^2$

$= 20$ M1 A1

19.

1 mark each for 1st, 2nd, 4th circled, less one mark for each other circled A3

20.

$\left(3\sqrt{5}-1\right)^2+\left(3+\sqrt{5}\right)^2=AB^2$ MA1

$9\sqrt{25}-3\sqrt{5}-3\sqrt{5}+1+9+3\sqrt{5}+3\sqrt{5}+\sqrt{25}$ MA1

$45-6\sqrt{5}+1+9+5+6\sqrt{5}=AB^2$ MA1

$60=AB^2$

$AB=\sqrt{60}$ A1

$AB=2\sqrt{15}$ A1

21.

$\frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} + \frac{\sqrt{3}(\sqrt{27} - 8)}{6}$ M1

$\frac{4\sqrt{3}}{3} + \frac{\sqrt{3}(\sqrt{27} - 8)}{6}$ A1

$\frac{4\sqrt{3}}{3} + \frac{9}{6} - \frac{8\sqrt{3}}{6}$ MA1

$\frac{3}{2}$ A1

Alternative

$\frac{24 + 3(\sqrt{27} - 8)}{6\sqrt{3}}$ MA1

$\frac{3\sqrt{27}}{6\sqrt{3}}$ A1

$\frac{3\sqrt{27}}{6\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ M1

$\frac{27}{18} = \frac{3}{2}$ A1

22.

$3^2 - 2^2 = 5$ M1

Base = $\sqrt{5}$ A1

Cos x = $\frac{\sqrt{5}}{3}$ tan x = $\frac{2}{\sqrt{5}}$ MA1

$\frac{\sqrt{5}}{3} + \frac{2\sqrt{5}}{5} = \frac{5\sqrt{5}}{15} + \frac{6\sqrt{5}}{15} = \frac{11\sqrt{5}}{15}$ M1 A1

23.

$AB^2 = 11^2 - (\sqrt{19})^2$ M1

$AB^2 = 102$ A1

$AB = \sqrt{102}$ A1

24.

$d^2 = (3 + 2\sqrt{3})^2 + (3 + \sqrt{3})^2 + (9 - \sqrt{3})^2$ MA1

$d^2 = 9 + 12\sqrt{3} + 12 + 9 + 6\sqrt{3} + 3 + 81 - 18\sqrt{3} + 3$ MA2

$d^2 = 117$ MA1

$d = 3\sqrt{13}$ A1

25.

$\frac{3}{4\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$ M1

$= \frac{3\sqrt{5}}{20}$

$a = \frac{3}{20}$ A1

26.

$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 7 = 220$	MA1
$r^2 = 30$	MA1
$r = \sqrt{30}$	A1

27.

$4 + 4a\sqrt{3} + 3a^2 = b + 20\sqrt{3}$	M1A1
$a = 5$	A1
$b = 4 + 75 = 79$	A1

28.

$x^2 + x^2 + x^2 = 81$	MA1
$x^2 = 27$ so $x = \sqrt{27}$	MA1
$3\sqrt{3}$	A1

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29. **(a)** 1.08×10^{-4} A1

(b) 4×10^{3} or 4000 A1

(c) $99x = 72$ MA1
$x = \frac{8}{11}$ A1

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30. $100x = 21.71717171...$ or $1000x = 217.17171717...$
$x = 0.21717171...$ $10x = 2.17171717...$

$99x = 21.5$ or $990x = 215$ or $9999x = 2171.5$ MA1

$x = \frac{215}{990}$ or $\frac{43}{198}$ or $\frac{21715}{99990}$ A1

31.

	Alternative
$100r = 31.8181818$	$1000r = 318.181818$
$-r = 0.3181818$	$-10r = 3.181818$
$99r = 31.5$	$990r = 315$
$r = \frac{315}{990}\left(\frac{7}{22}\right)$	$r = \frac{315}{990}\left(\frac{7}{22}\right)$

MA1

A1

32.

$x = 0.3727272...$

$10x = 3.727272...$ **or** $100x = 37.2727272...$

$1000x = 372.727272 ...$ M1

$990x = 369$ **or** $99x = 36.9$

$\left(\text{or } x = \frac{369}{990}\right)$ $\left(\text{or } x = \frac{36.9}{99}\right)$ A1

$\frac{41}{110}$ A1

33.

$x = 0.561561\ldots$
$1000x = 561.561\ldots$
$999x = 561$ M1
$x = \frac{187}{333}$ A1

34.

A rational because area = 24 A1
B irrational because π irrational A1
C irrational because $\sqrt{19}$ irrational A1

35.

(a) rational because 6 is rational A1

(b) irrational because π or $3\sqrt{\pi}$ is irrational A1

(c) irrational because $\sqrt{8}$ is irrational A1

36.

$100x = 8.5858585...$
$x = 0.085858...$
$99x = 8.5$

$\frac{85}{990}$ or $\frac{17}{198}$ M1 A1

37.

rational	irrational
irrational	rational
irrational	irrational

A4

allow A1 for 3 correct, A2 for 4 correct, A3 for 5 correct

38.

correct use of Pythagoras' theorem M1

a is rational because $a = 7$ A1

b is irrational because $b = \sqrt{74}$ A1

$(2\sqrt{11} - 3)^2 + (6 + \sqrt{11})^2 = 44 - 12\sqrt{11} + 9 + 36 + 12\sqrt{11} + 11 = 100$ M1 A1

c is rational because $c = 10$ A1

39.

$x = 0.08181818181...$

$100x = 8.18181818181\ldots$

$99x = 8.1$

$x = \frac{8.1}{99}$

$\frac{81}{990}$ or $\frac{9}{110}$ M1 A1

Alternative

$10x = 0.8181818181...$

$1000x = 81.8181818181\ldots$

$990x = 81$

$x = \frac{81}{990}$ or $\frac{9}{110}$

40.

$(7-\sqrt{5})(7+\sqrt{5}) = 7^2 - 5 = 44$	rational	M1A1
$[(7-\sqrt{5})-(7+\sqrt{5})]^2 = (-2\sqrt{5})^2 = 20$	rational	M1A1
$\frac{14\sqrt{10}}{7\sqrt{5}} = 2\sqrt{5}$	irrational	A1

41.

$V = 4000\,(0.9995)^2$	A1

Q1

A piece of land in the shape of a rectangle has a length of 8.3×10^6 centimetres and a width of 3.7×10^4 centimetres.

(a) Work out the area of the land, giving your answer in standard form.

Answer _____ cm^2 [1]

(b) Work out the perimeter of the land, giving your answer in standard form.

Answer _____ cm [2]

Q2 Write the following in standard form.

(a) 0.00000385

Answer _____ [1]

(b) 167×10^{-9}

Answer _____ [1]

Q3 $(2.5 \times 10^6) \times (4.8 \times 10^x) = A \times 10^4$

Find the values of A and x

Answer A = _____ , x = _____ [3]

Q4

(a) Calculate $(3.6 \times 10^5) \times (4.9 \times 10^{-4})$ giving your answer in standard form.

Answer _____ [1]

(b) $(2.5 \times 10^6) \times (4.8 \times 10^x) = A \times 10^4$

Find the values of A and x

Answer A = _____ , x =_____ [3]

Q5 **Show clearly** how one solution to the equation $2x - \frac{12}{x} = 0$ is $\sqrt{6}$

[2]

Q6

£1000 is invested at 2% per annum compound interest.

(a) Circle the formula which gives the value of the investment after n years.

$V = (1000(1.2))^n$ $V = 1000(1.2)^n$ $V = (1000(1.02))^n$ $V = 1000(1.02)^n$

[1]

(b) Calculate the compound interest earned on £1000 invested at 2% per annum for 8 years.

Answer £ _____ [2]

Q7

(a) £8000 is invested at 3% per annum compound interest.

Complete the formula for the amount £A after n years.

$A = 8000(\_\_\_\_\_)^n$ [1]

(b) Calculate the total **interest** earned after 4 years.

Answer £ _____ [2]

1.

(a) 3.071×10^{11} A1

(b) 1.6674×10^{7} A2
(award A1 for 8.337×10^{6}) (adding 2 sides, not doubling)

2.

(a) 3.85×10^{-6} A1

(b) 1.67×10^{-7} A1

3.

$2.5 \times 4.8 = 12$ so A = 1.2 MA1

$12 \times 10^{6+x} = 12000$ **or** $6 + x + 1 = 4$ or $\dfrac{12000}{2.5 \times 10^{6}} = 4.8 \times 10^{-3}$ MA1

so $x = -3$ A1

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4. **(a)** 1.764×10^2 A1

(b) $2.5 \times 4.8 = 12$ so A = 1.2 MA1

$12 \times 10^{6+x} = 12000$ **or** $6 + x + 1 = 4$ or $\frac{12000}{2.5 \times 10^6} = 4.8 \times 10^{-3}$ MA1

so $x = -3$ A1

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5. $2x = \frac{12}{x}$

$2x^2 = 12$ MA1

$x^2 = 6$

$x = \sqrt{6}$ MA1

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6. **(a)** $V = 1000\ (1.02)^n$ A1

(b) $1000(1.02)^8 = 1171.65(9381)$ MA1

171.65 or 171.66 MA1

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7. **(a)** 1.03 A1

(b) $8000\ (1.03)^4 = 9004.07$ MA1

1004.07 A1
