St. Patrick's High School, Keady

Mathematics Department

GCSE Mathematics Practice Booklet

M8

Topic 5 – Algebra 2

Equation of a Circle

Equation of tangents to a circle

Simultaneous Equations (Graphically)

Quadratic Graphs (including intersection with y= mx + c)

Cubic, Reciprocal and exponential functions

Conversion Graphs

Questions taken from CCEA Past Papers

Keady Maths

Q1

(a) The point $(5, a)$ lies on the circle $x^2 + y^2 = 50$

Find the possible values of a.

Answer _____ [2]

(b) Find the equation of the tangent to the circle $x^2 + y^2 = 50$ at $(-1, 7)$.

Answer _____ [3]

Q2 A (8, −6) and B (−8, 6) are ends of the diameter AB of a circle.

(a) What is the centre of the circle?

Answer _____ [1]

(b) What is the equation of the circle?

Answer _____ [1]

(c) What is the gradient of the diameter AB?

Answer _____ [1]

(d) Find the equation of the tangent to the circle at A.

Answer _____ [3]

(e) Write down the equations of the two tangents to this circle which are parallel to the y axis.

Answer _____ and _____ [2]

(f) Find the points of intersection of these tangents with the tangent at A.

Answer _____ and _____ [2]

Q3 P (2, 3) lies on the circumference of the circle $x^2 + y^2 = a^2$

(a) Work out the value of a, giving your answer in surd form.

Answer _____ [2]

(b) Work out the equation of the tangent to this circle at P.

Answer _____ [4]

(c) Work out the equation of the other tangent to this circle which is parallel to the tangent at P.

Answer _____ [3]

Q4 P is a point on the circle $x^2 + y^2 = 169$

P also lies on the line $y = 12$

(a) Show that the radius to P has gradient $\pm \frac{12}{5}$

[3]

(b) Hence find the two possible equations for the tangent at P.

Answer _____ , _____ [3]

Q5 $(-3, 4)$ is a point on the circle $x^2 + y^2 = 25$

(a) Show that the equation of the tangent to the circle at this point is $4y = 3x + 25$

[4]

(b) Find the coordinates of the points of intersection of this tangent and the curve $y = x^2 + 6$

Answer _____ [6]

Q6 $y = \frac{3}{4}x + c$ is a tangent at the point P to the circle $x^2 + y^2 = 100$, centre O at $(0, 0)$.

(a) Write down the gradient of the radius OP.

Answer _____ [1]

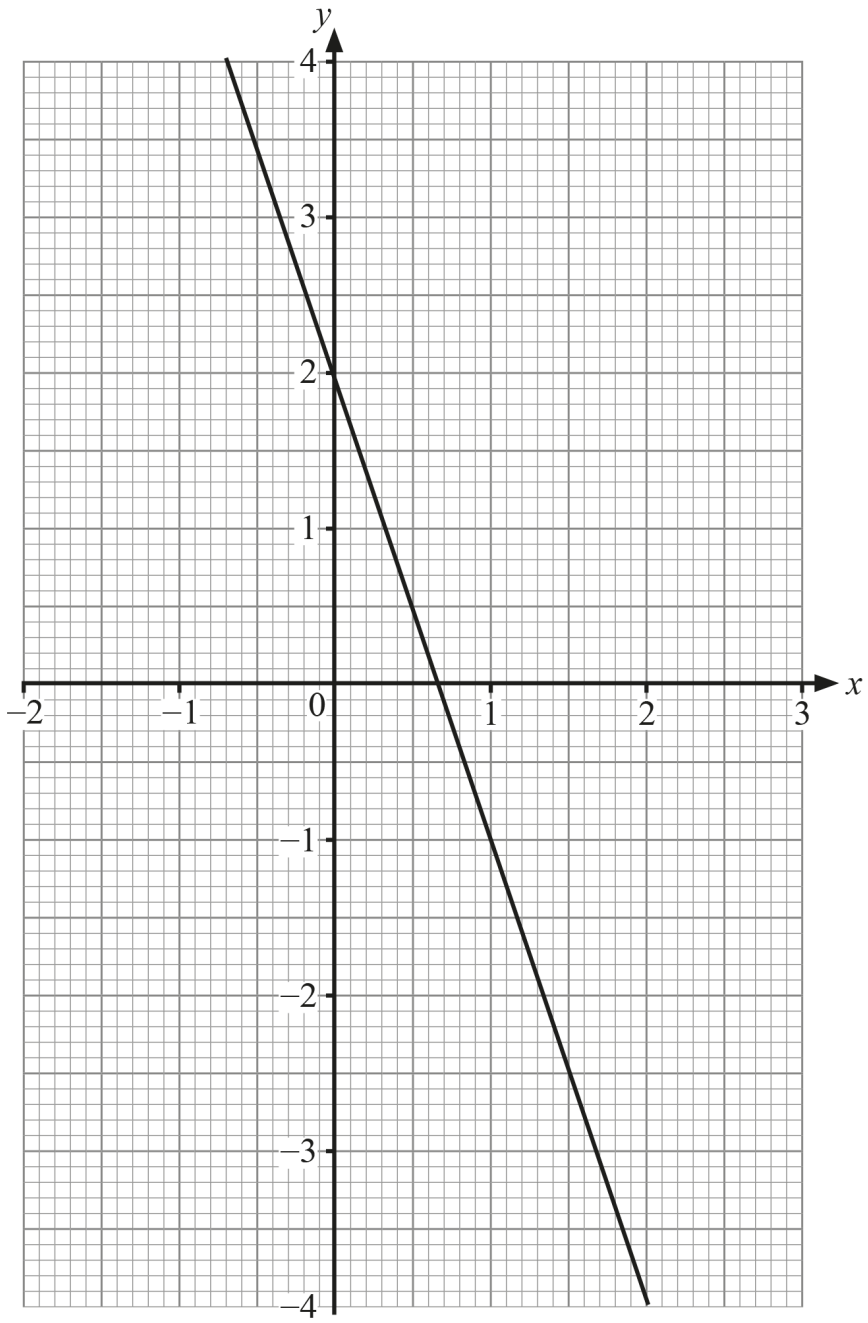
(b) Write down the equation of the line OP.

Answer _____ [1]

(c) Hence find the possible coordinates of P.

Answer _____ or _____ [4]

Q7



By drawing a suitable line on the grid opposite solve the simultaneous equations

$y = 2x - 2$

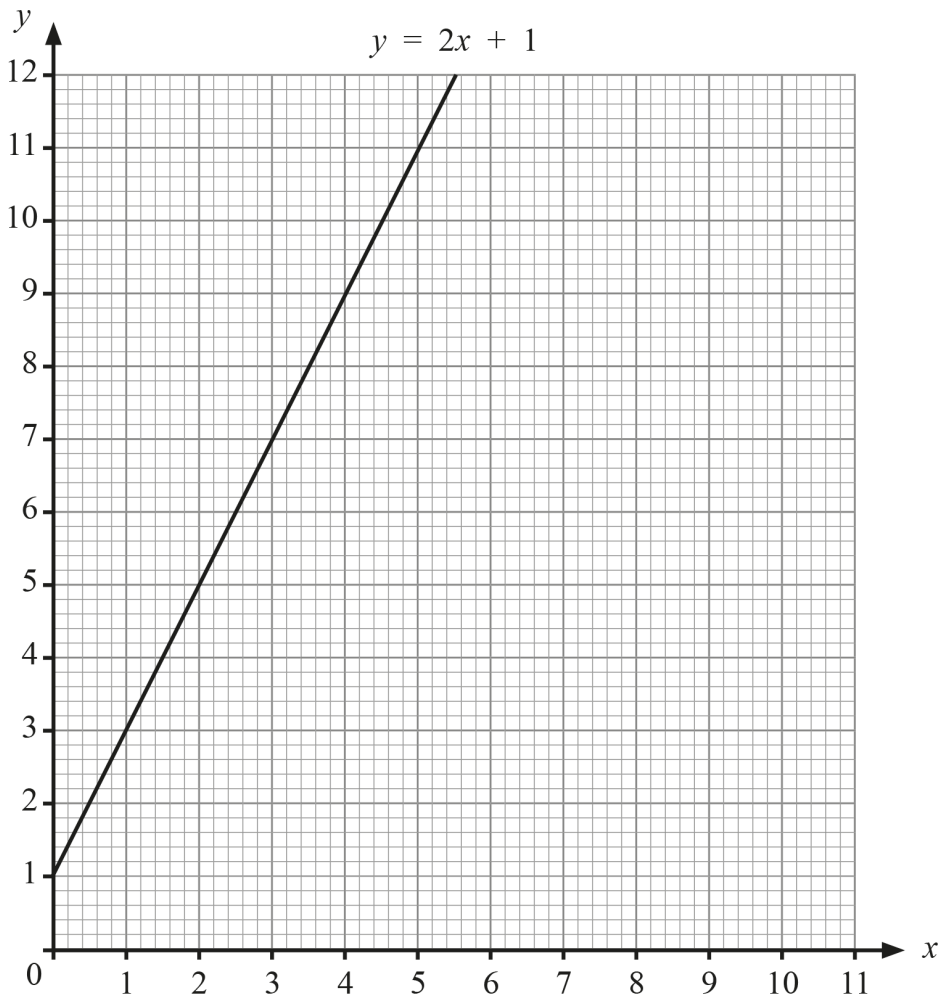
$y = -3x + 2$

Answer $x =$ _____ $y =$ _____ [4]

Q8 Use graphs to solve the simultaneous equations

$$y = 2x + 1 \quad \text{and} \quad y = 10 - x$$

The graph of $y = 2x + 1$ has already been drawn for you.

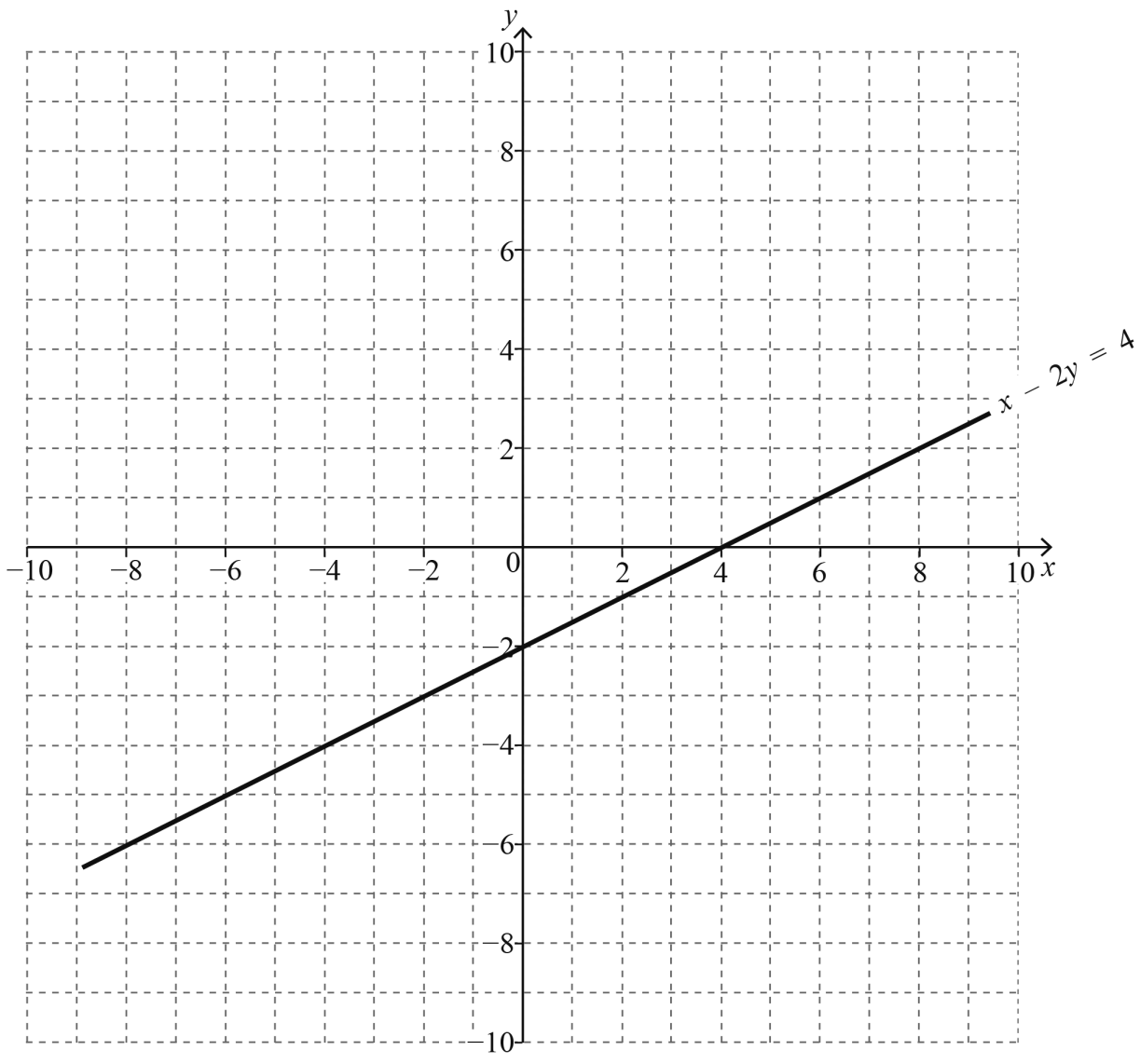


Answer $x =$ _____ and $y =$ _____ [4]

Q9 By drawing a suitable line on the grid, solve the simultaneous equations

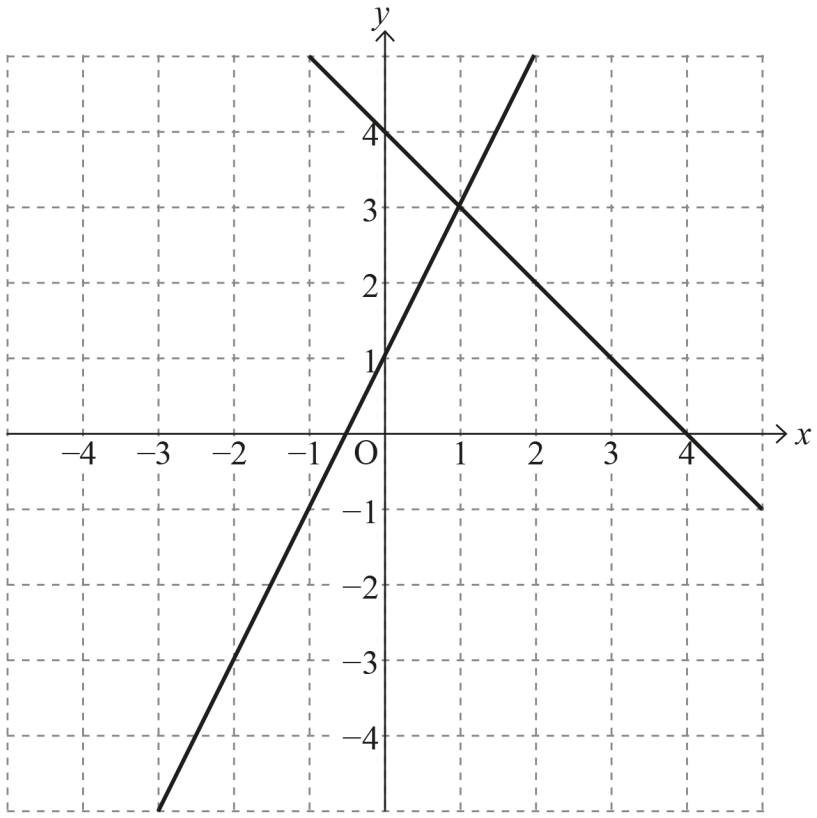
$$x - 2y = 4$$

$$y = 3x + 3$$



Answer $x =$ _____ $y =$ _____ [4]

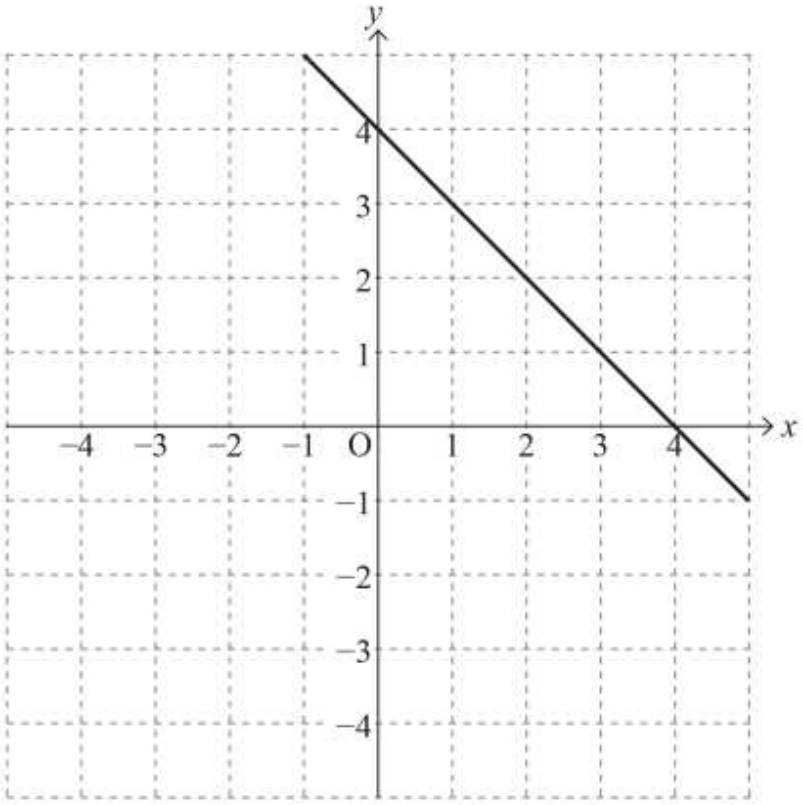
Q10 **(a)**



The lines $y = 2x + 1$ and $x + y = 4$ have been drawn on the grid.

By drawing another line on the grid above, indicate clearly by the letter R the region satisfying $y \geq 2x + 1$ and $x + y \leq 4$ and $x \geq -1$ [2]

(b)



By drawing more lines on the grid above, indicate clearly by the letter B the region satisfying $y \leq 2x$ and $x + y \leq 4$ and $y \geq 1$ [2]

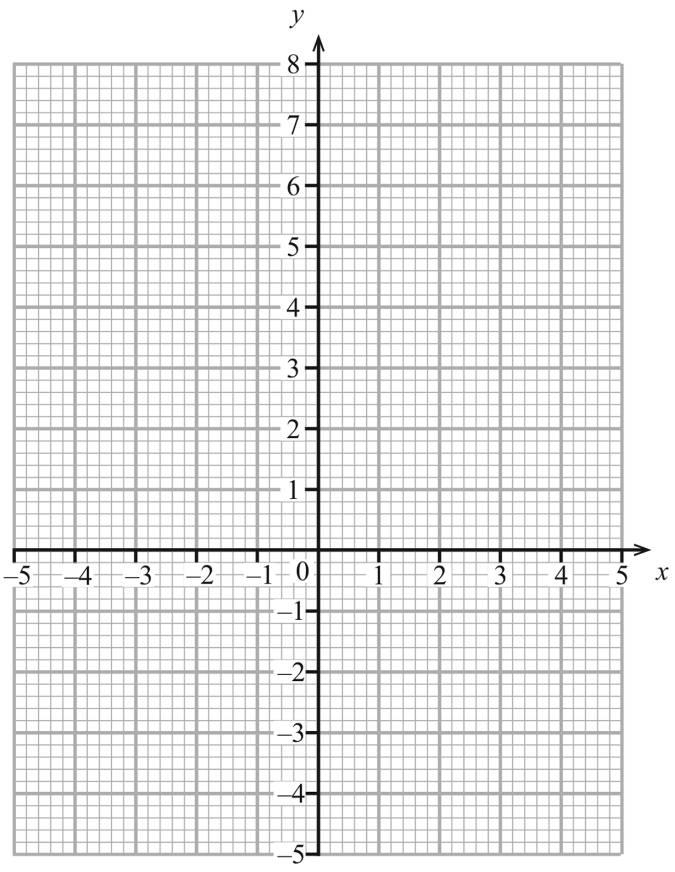
Q11 Part of the table for the graph of $y = x^2 - 2x - 3$ is shown below.

(a) Fill in the blanks in the table.

x	–2	–1	0	1	2	3	4
y	5	0			–3	0	5

[2]

(b) Use the values from the table to draw the graph.



[2]

(c) By drawing an appropriate line on the graph, solve the equation

$$x^2 - 2x - 3 = x - 1$$

Answer $x =$ _____ [3]

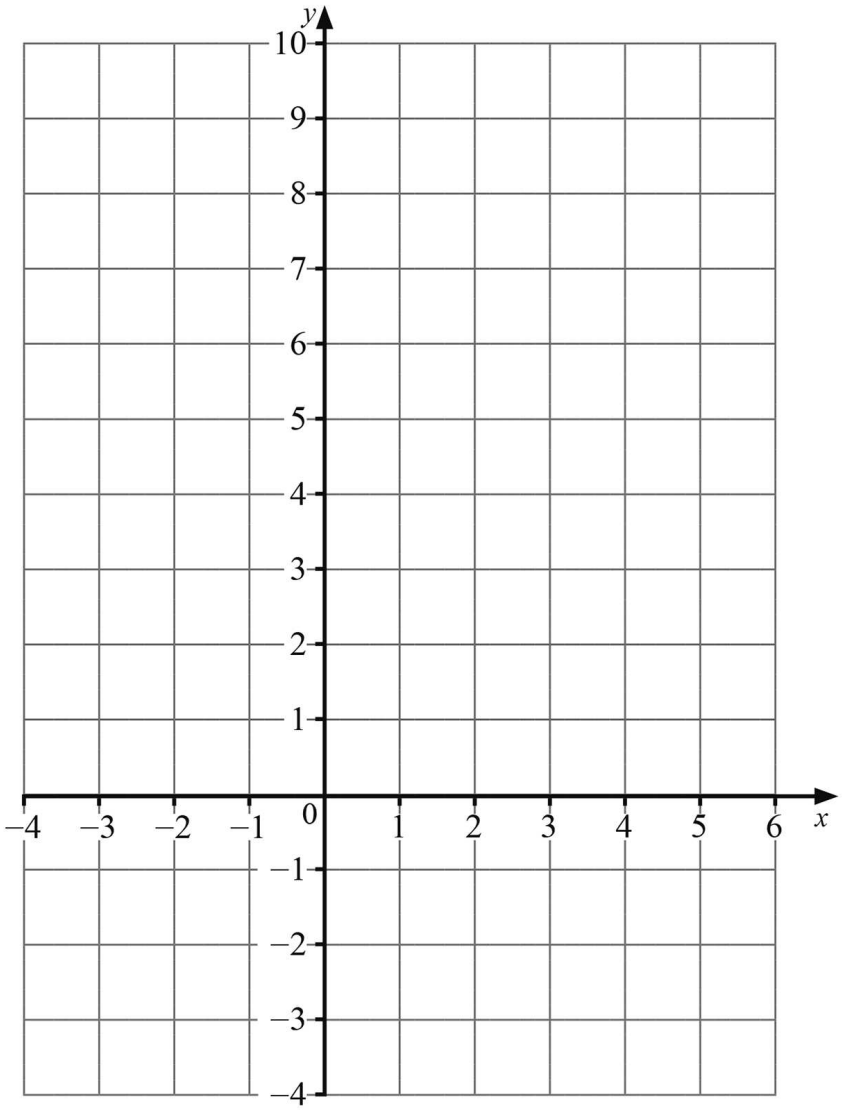
Q12 Part of the table for the graph of $y = x^2 - 2x - 1$ is shown below.

(a) Fill in the blanks in the table.

x	-2	-1	0	1	2	3	4
y	7			-2		2	7

[2]

(b) Use the values from the table to draw the graph of $y = x^2 - 2x - 1$ for $-2 \leqslant x \leqslant 4$

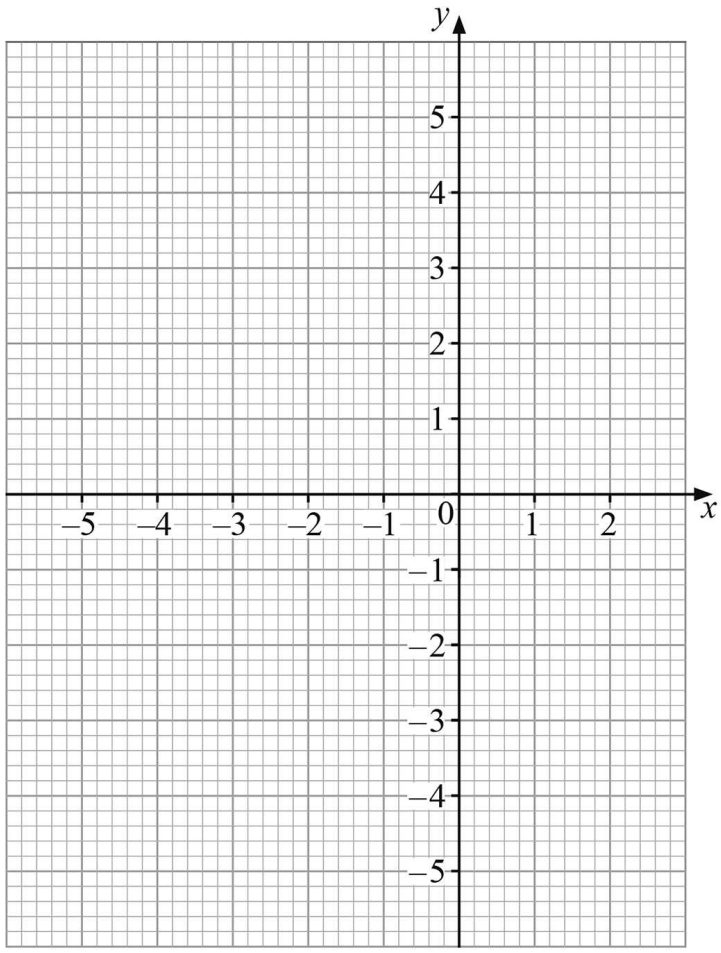


[2]

Q13 Here is a table of values for $y = 1 - 3x - x^2$

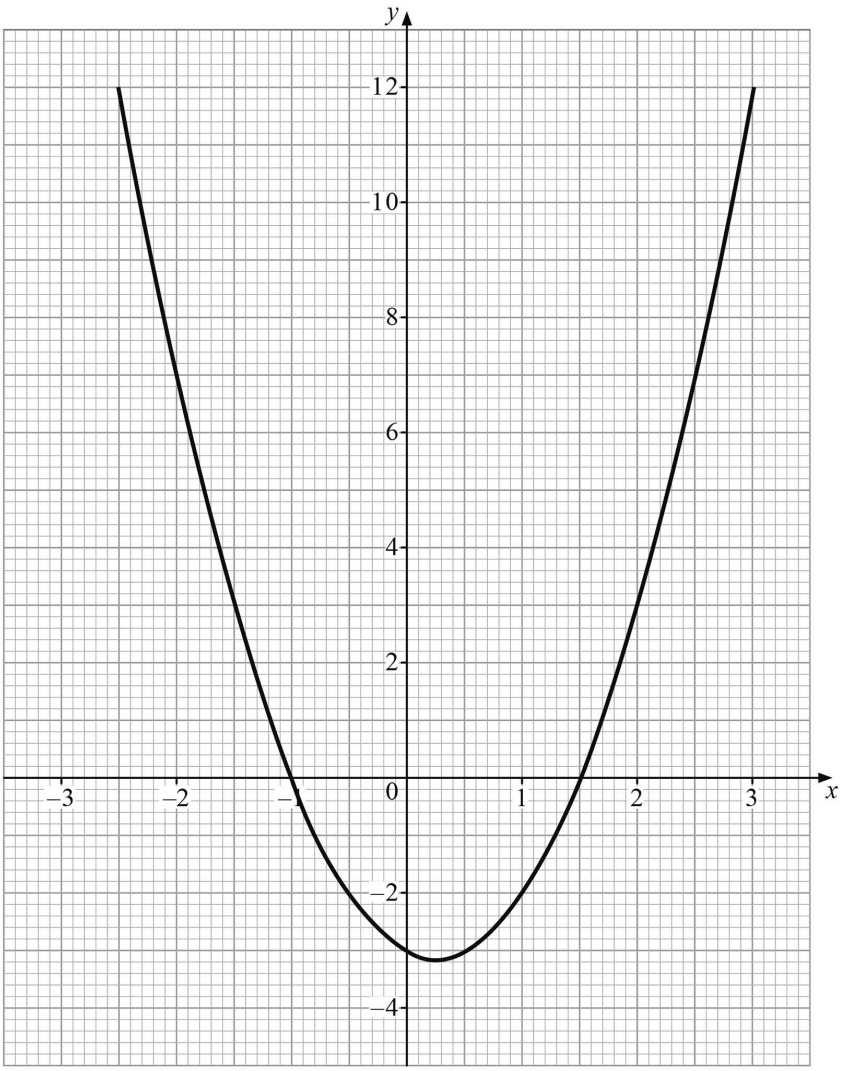
x	–4	–3	–2	–1	0	1
y	–3	1	3	3	1	–3

Use the table to draw the graph of $y = 1 - 3x - x^2$ on the grid below for values of x from –4 to 1



[2]

Q14 The graph of $y = 2x^2 - x - 3$ for $-2.5 \leqslant x \leqslant 3$ is shown below.



Use the graph to solve the equation

$$2x^2 - x - 3 = 3 - 1.5x$$

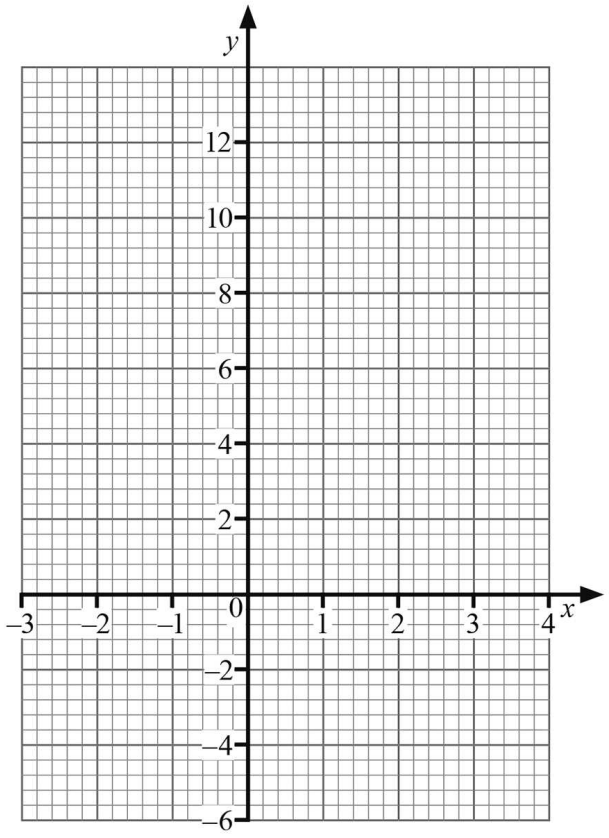
Answer $x =$ _____ [3]

Q15 **(a)** Complete the table below for $y = 2x^2 - x - 3$

x	–2	–1	0	1	2	3
y		0	–3	–2	3	12

[1]

(b) On the grid draw the graph of $y = 2x^2 - x - 3$ for $x = -2$ to $x = 3$



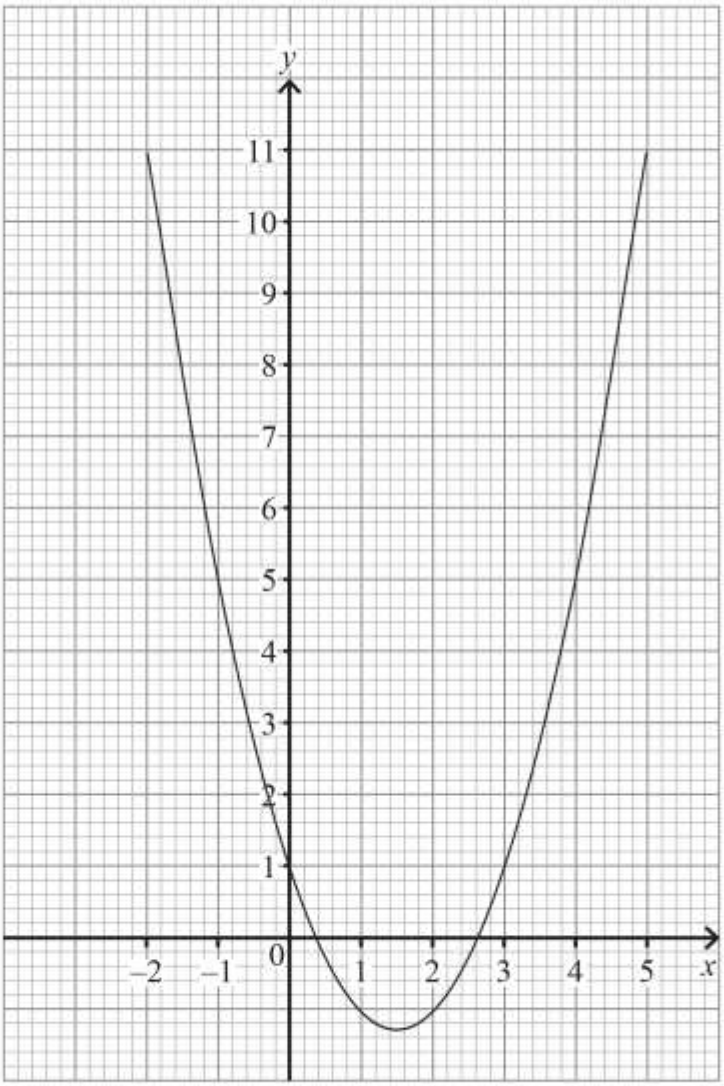
[2]

Q16 Tim has drawn the graph of $y = 2x^2 - x - 23$

What line should be drawn on the graph to solve the equation $x^2 - x - 12 = 0$?

Answer _____ [2]

Q17 On the grid is drawn the graph of $y = x^2 - 3x + 1$



By drawing appropriate lines, use this graph to

(a) find the gradient of the curve at the point (2, −1)

Answer _____ [2]

(b) solve $x^2 - 3x - 3 = 0$

Answer x = _____ [2]

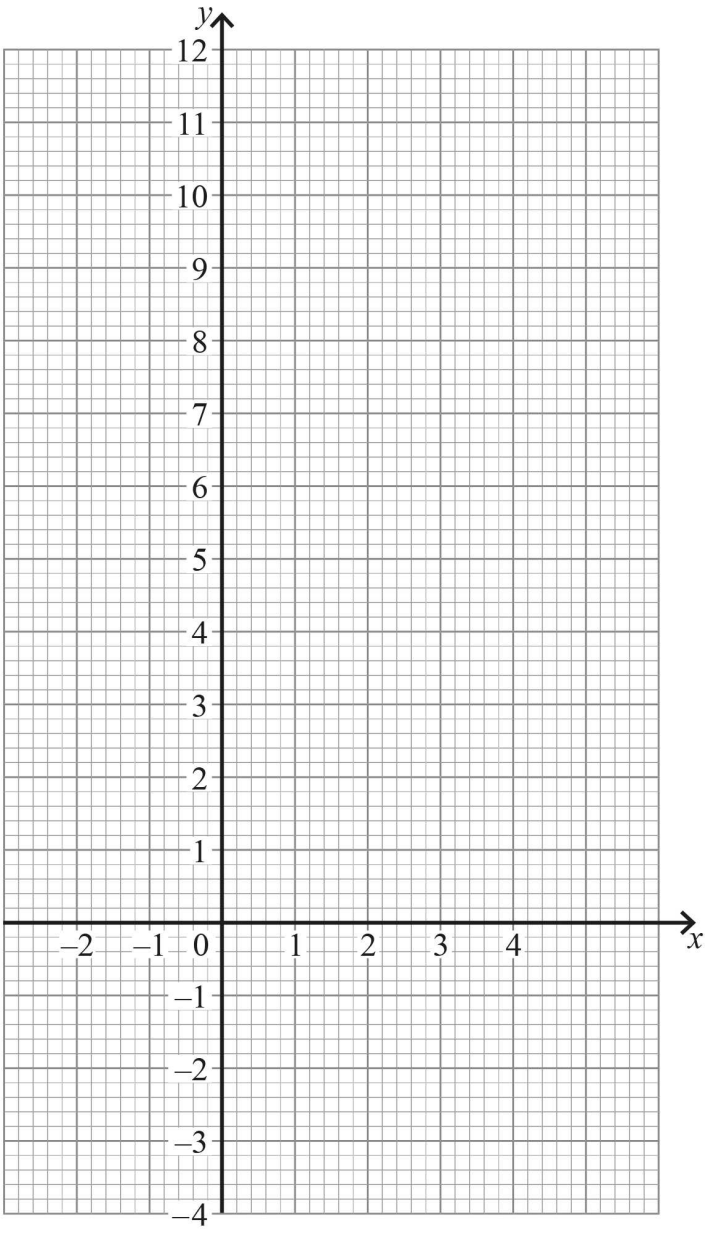
(c) solve $x^2 - 4x - 1 = 0$

Answer x = _____ [3]

Q18 The following table gives some values for the quadratic equation $y = x^2 - 3x + 1$

x	–2	–1	0	1	2	3	4
y	11	5	1	–1	–1	1	5

(a) On the grid below, draw the graph of $y = x^2 - 3x + 1$ for values of x between –2 and 4



[2]

(b) Use your graph to estimate the values of x for which $y = 3$

Answer $x =$ _____ [2]

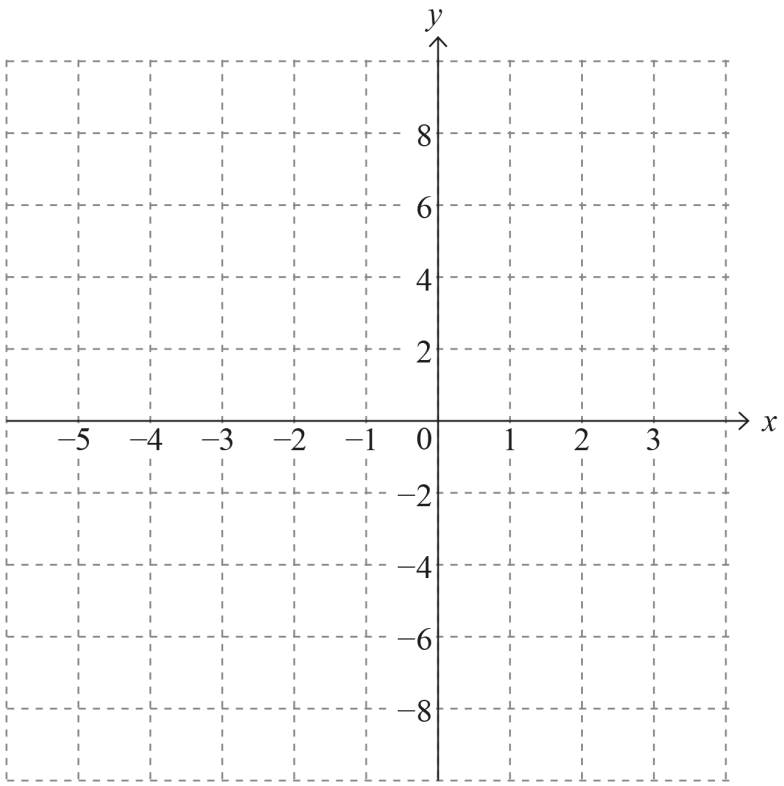
Q19

(a) Complete the table for $y = x^2 + 3x - 3$

x	−4	−3	−2	−1	0	1	2
y	1		−5	−5	−3	1	

[2]

(b) Draw the graph of $y = x^2 + 3x - 3$ from $x = -4$ to $x = 2$ [2]



(c) Use your graph to estimate the solutions to $x^2 + 3x - 3 = 0$

Answer _____ [2]

(d) (i) By drawing a line on the grid, solve $x^2 + 3x - 3 = x + 1$

Answer _____ [2]

(ii) Write down in simplest form the quadratic equation that has been solved in **(i)**.

Answer _____ [1]

(e) What line would be drawn on the grid to solve $x^2 + 4x - 6 = 0$?

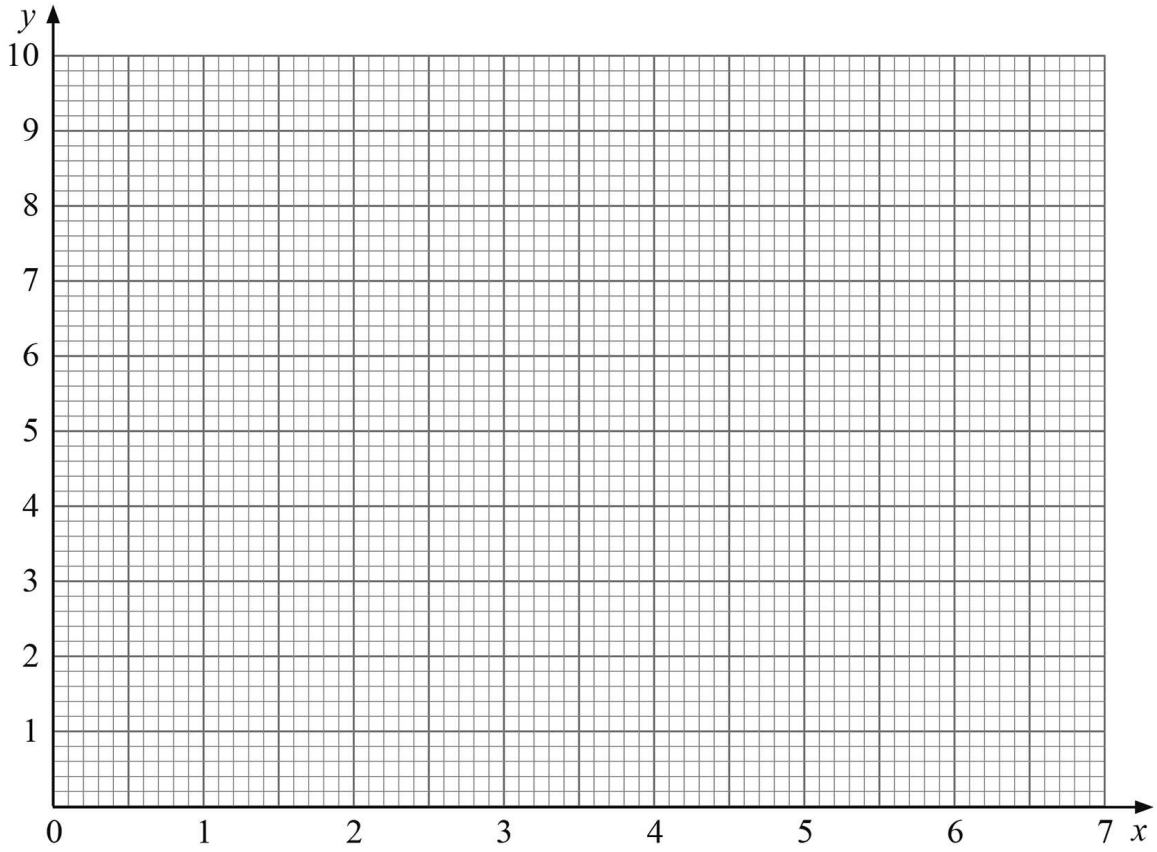
Answer _____ [2]

Q20 **(a)** Complete the table of values for $y = \frac{3}{x}$

x	0.5	1	2	3	4	5	6
y		3		1			

[2]

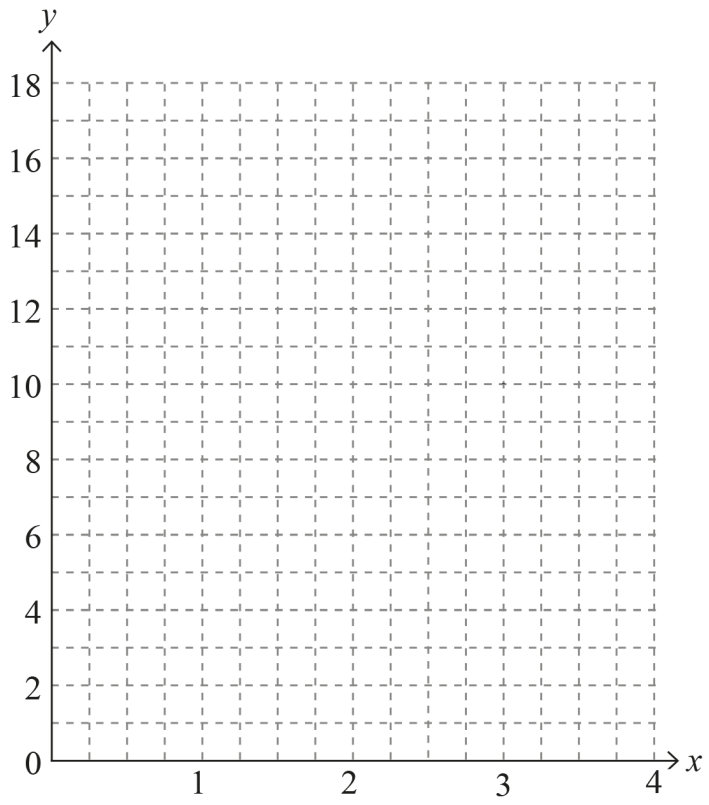
(b) On the grid below, draw the graph of $y = \frac{3}{x}$ for $0.5 \leqslant x \leqslant 6$



[2]

Q21

(a) Sketch the curve $y = 2^x$ on the grid below for $0 \leq x \leq 4$



[3]

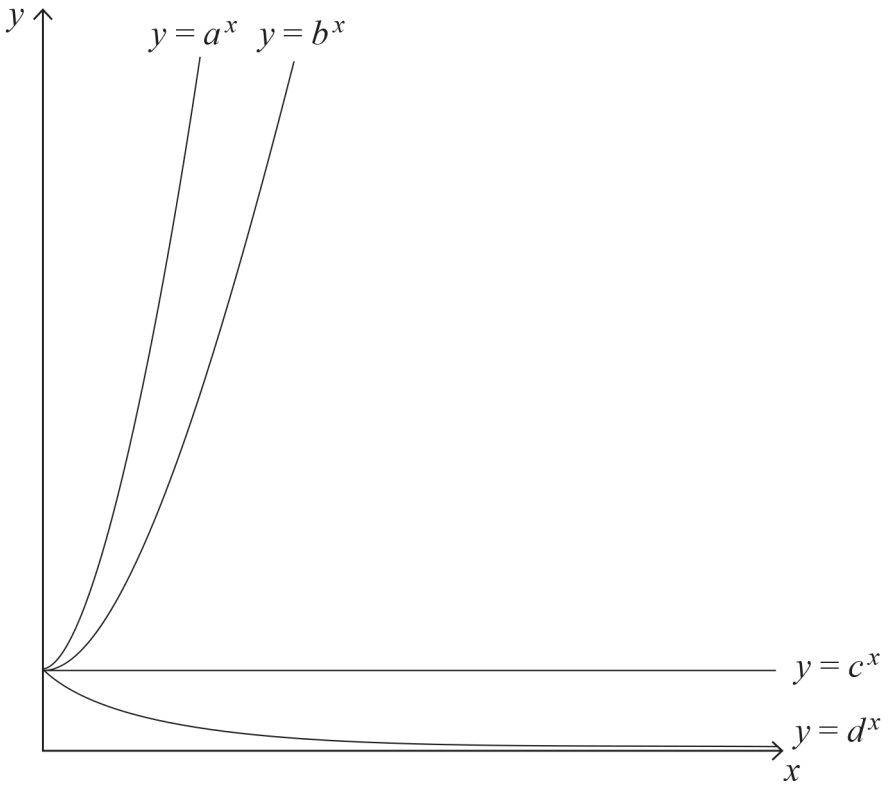
(b) In **(a)**, if £y represents the value of one share in an investment company x years after purchase, explain in words what is happening to the value.

Answer _____ [1]

(c) Use your graph to predict after how many **months** the value of the share will be 10 times greater than the original purchase value.

Answer _____ months [2]

Q22



Sketches of $y = a^x, y = b^x, y = c^x, y = d^x$ are drawn above.

(a) a and b are numbers. One has the value 2 and the other has the value 3

Which is which?

Answer a =_____ , b = _____ [1]

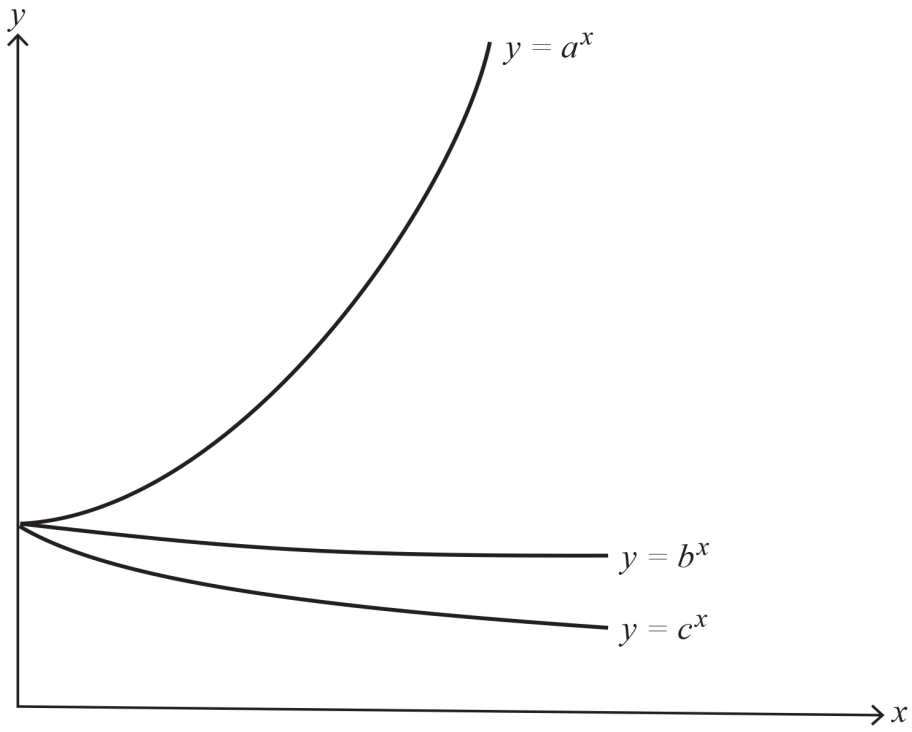
(b) Write down the value of c

Answer c = _____ [1]

(c) Estimate the value of d

Answer d = _____ [1]

Q23



a, b and c are three decimal numbers.

The graphs of $y = a^x$, $y = b^x$, $y = c^x$ are sketched above.

One of a, b, c lies between 0.2 and 0.6,
one of a, b, c lies between 0.8 and 1.2,
and one of a, b, c lies between 1.4 and 1.8

(a) Which of a, b, c lies between 0.2 and 0.6?

Answer _____ [1]

(b) Which of a, b, c lies between 0.8 and 1.2?

Answer _____ [1]

(c) d is a decimal which lies between 2.0 and 2.4

Draw one possible sketch of the graph $y = d^x$ on the grid above. [1]

1.

(a) $5^2 + a^2 = 50$ so $a^2 = 25$ MA1

5 or –5 A1

(b) Point (–1,7) grad radius –7 MA1

Grad tangent $\frac{1}{7}$ $y = \frac{1}{7}x + c$ MA1

$7 = -\frac{1}{7} + c$

$y = \frac{1}{7}x + \frac{50}{7}$ A1

2.

(a) (0, 0) A1

(b) $x^2 + y^2 = 100$ A1

(c) $-\frac{3}{4}$ A1

(d) gradient $\frac{4}{3}$ MA1

$y = \frac{4}{3}x + c$ through (8,–6) $-6 = \frac{32}{3} + c$ MA1

$y = \frac{4}{3}x - \frac{50}{3}$ A1

(e) $x = 10, x = -10$ A1A1

(f) $(10, -\frac{10}{3})$, (–10, –30) A1 A1

3.

(a) $2^2 + 3^2 = 13$ so $a = \sqrt{13}$ M1 A1

(b) gradient of radius $= \frac{3}{2}$ MA1

gradient of tangent $= -\frac{2}{3}$ MA1

$y = -\frac{2}{3}x + c$; $3 = -\frac{4}{3} + c$ MA1

$y = -\frac{2}{3}x + \frac{13}{3}$ A1

(c) other end of diameter is $(-2, -3)$ MA1

$y = -\frac{2}{3}x + c$; $-3 = \frac{4}{3} + c$ MA1

$y = -\frac{2}{3}x - \frac{13}{3}$ A1

(Alternative $y = -\frac{2}{3}x - \frac{13}{3}$ (using symmetry) A3

4.

(a) $x^2 + 144 = 169$

$x = 5$ or -5 MA1

P is $(5, 12)$ or $(-5, 12)$ MA1

Gradient of radius is $\frac{12}{5}$ or $-\frac{12}{5}$ A1

(b) Gradient of tangents $-\frac{5}{12}$ or $\frac{5}{12}$ MA1

Equations $y = -\frac{5}{12}x + c$, using $(5, 12)$ gives $c = \frac{169}{12}$

$y = -\frac{5}{12}x + \frac{169}{12}$ MA1

and $y = \frac{5}{12}x + c$, using $(-5, 12)$ gives $c = \frac{169}{12}$

$y = \frac{5}{12}x + \frac{169}{12}$ MA1

5.

(a) Grad radius $= -\frac{4}{3}$ MA1

Grad tangent $= \frac{3}{4}$ MA1

$y = \frac{3}{4}x + c,\ 4 = \frac{3}{4} \times (-3) + c,\ c = \frac{25}{4}$ MA1

$y = \frac{3}{4}x + \frac{25}{4},\ 4y = 3x + 25$ MA1

(b) $4(x^2 + 6) = 3x + 25$ MA1

$4x^2 + 24 = 3x + 25$

$4x^2 - 3x - 1 = 0$ MA1

$(4x + 1)(x - 1) = 0$ MA1

$x = -\frac{1}{4}$ or 1 MA1

$y = 6\frac{1}{16}$ or 7 MA1

$\left(-\frac{1}{4}, 6\frac{1}{16}\right), (1, 7)$ MA1

6.

(a) $-\frac{4}{3}$ A1

(b) $y = -\frac{4}{3}x$ A1

(c) $x^2 + \frac{16}{9}x^2 = 100$ MA1

$\frac{25}{9}x^2 = 100$ MA1

$x = 6$ **or** -6 MA1

$(6, -8)$ **or** $(-6, 8)$ MA1

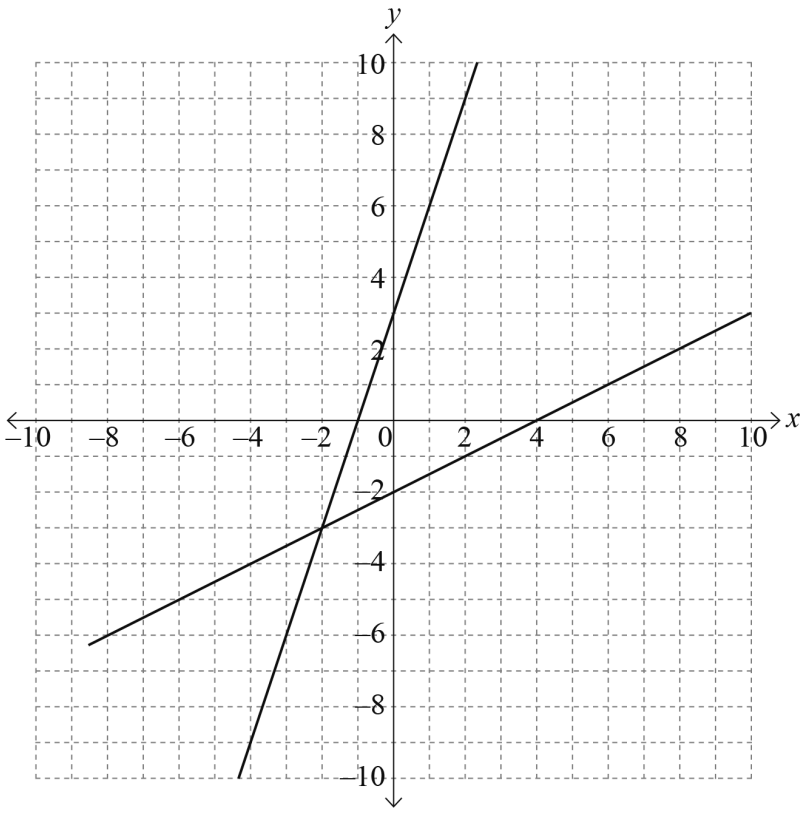
7.

Correct line drawn	M1 A1
$x = 0.8$ $y = -0.4$	A1 A1
Correct line drawn	M1 A1
$x = 0.8$ $y = -0.4$	A1 A1

8.

Line $x + y = 10$ passes through points (0, 10) and (10, 0)	M1 A1
$x = 3$ and $y = 7$	A1 A1

9.



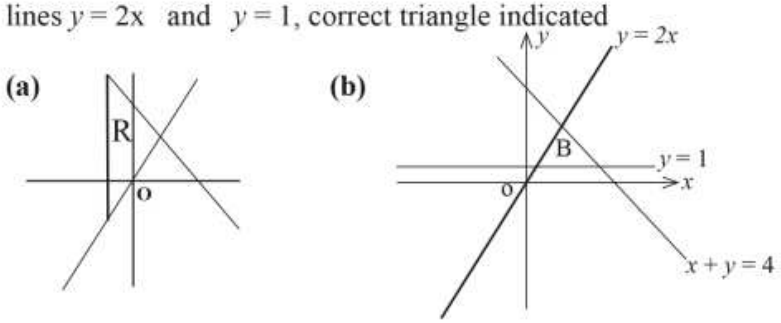
Correct line drawn, gradient 3, intercept (0,3) MA1 MA1

$x = -2 \quad y = -3$ A1 A1

10.

(a) line $x = -1$ drawn, correct triangle indicated MA1 A1

(b) lines $y = 2x$ and $y = 1$, correct triangle indicated MA1 A1



11.

(a) –3 and –4 A1 A1

(b) plot all points correctly A1
smooth curve through all the points A1

(c) line $y = x - 1$ correctly drawn MA1
answers from graph – 0.6 and 3.6 A1 A1

12.

(a) 2, –1, –1 A2
(A1 for correct values)

(b) Correct smooth curve drawn from $x = -2$ to $x = 4$ A2
(A1 for all 7 points in the candidates's table plotted correctly)

13.

All six points plotted correctly	A1
Smooth curve drawn through all six points	A1

14.

Drawing the graph of $y = 3 - 1.5x$	MA1
$x = -1.9$	A1
$x = 1.6$	A1

15.

(a)	7	A1
(b)	points plotted correctly	A1
	smooth curve through the points	A1

16.

$x^2 - x - 12 = 0 \Rightarrow 2x^2 - 2x - 24 \ (= 0)$	MA1
$y = x + 1$	MA1

17.

(a) Tangent drawn at (2, −1) MA1

Gradient from candidate's tangent, approximately 1 MA1

(b) Drawing $y = 4$, −0.8 and 3.8 M1 A1

(c) $x^2 - 4x - 1 = 0$ so $x^2 - 3x + 1 = x + 2$ M1

drawing $y = x + 2$, −0.2 and 4.2 MA1 A1
accept from –0.25 to –0.2 for negative value
in all readings from their drawn lines, accept a little tolerance

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18.

(a) All seven points plotted correctly A1

Correct smooth curve drawn A1

(b) –0.6, 3.6 A1 A1

19.

(a) –3 7 A1 A1

(b) points plotted, smooth curve A1 A1

(c) readings from graph (two intercepts) A1 A1

(d) **(i)** line $y = x + 1$ drawn, readings from graph (two values for x) MA1 A1

(ii) $x^2 + 2x - 4 = 0$ A1

(e) $x^2 + 3x - 3 = -x + 3$ MA1

$y = -x + 3$ A1

20.

(a)

x	0.5	1	2	3	4	5	6
y	**6**	3	**1.5**	1	**0.75**	**0.6**	**0.5**

all 5 values correct A2

(A1 at least 3 correct)

(b) Accurately drawn graph between $x = 0.5$ and $x = 6$ A2
(all 7 points plotted correctly gets A1)

21.

(a)	5 points plotted (0, 1), (1, 2), (2, 4), (3, 8), (4, 16)	A2
	Joined with smooth curve	A1
(b)	Doubling in value each year	A1
(c)	Reading from their graph at $y = 10$	A1
	Converting to months, approx. 39 or 40 months	A1

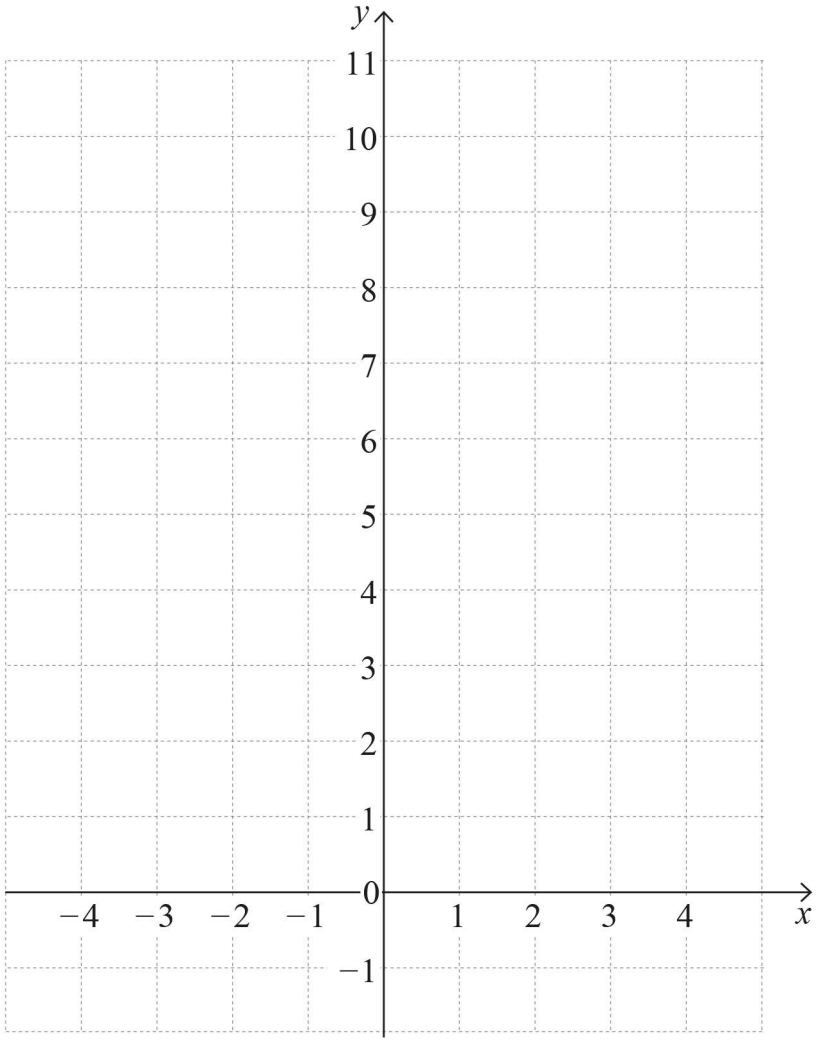
22.

(a)	$a = 3, b = 2$	A1
(b)	$c = 1$	A1
(c)	fraction around $\frac{1}{2}$	A1

23.

(a)	c	A1
(b)	b	A1
(c)	suitable sketch similar to $y = a^x$ above $y = a^x$	A1

Q1



(a) Draw the graph of $y = x^2 - 2x + 3$ for $-2 \leq x \leq 4$ on the grid above. [2]

(b) Use your graph to find the gradient of the curve when $x = 2$

Answer _____ [2]

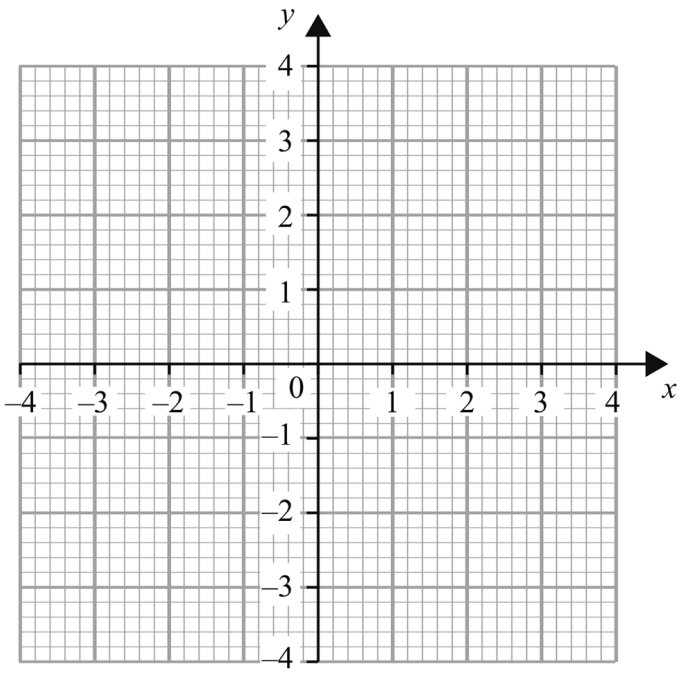
(c) By drawing an appropriate line solve $2x^2 - 4x - 1 = 0$

Answer _____ [2]

(d) What line would you draw on your graph to solve the equation $x^2 + 12x + 4 = 0$?

Answer _____ [2]

Q2 **(a)** Draw the graph of $y = x^2 - x - 2$ for values of x from –2 to 3 [2]



(b) **(i)** Write down the equation of the line of symmetry of the curve.

Answer _____ [1]

(ii) Hence **calculate** the minimum value of the curve.

Answer _____ [1]

Q3

(a) Complete the table for $y = 2x^2 - 4x - 5$

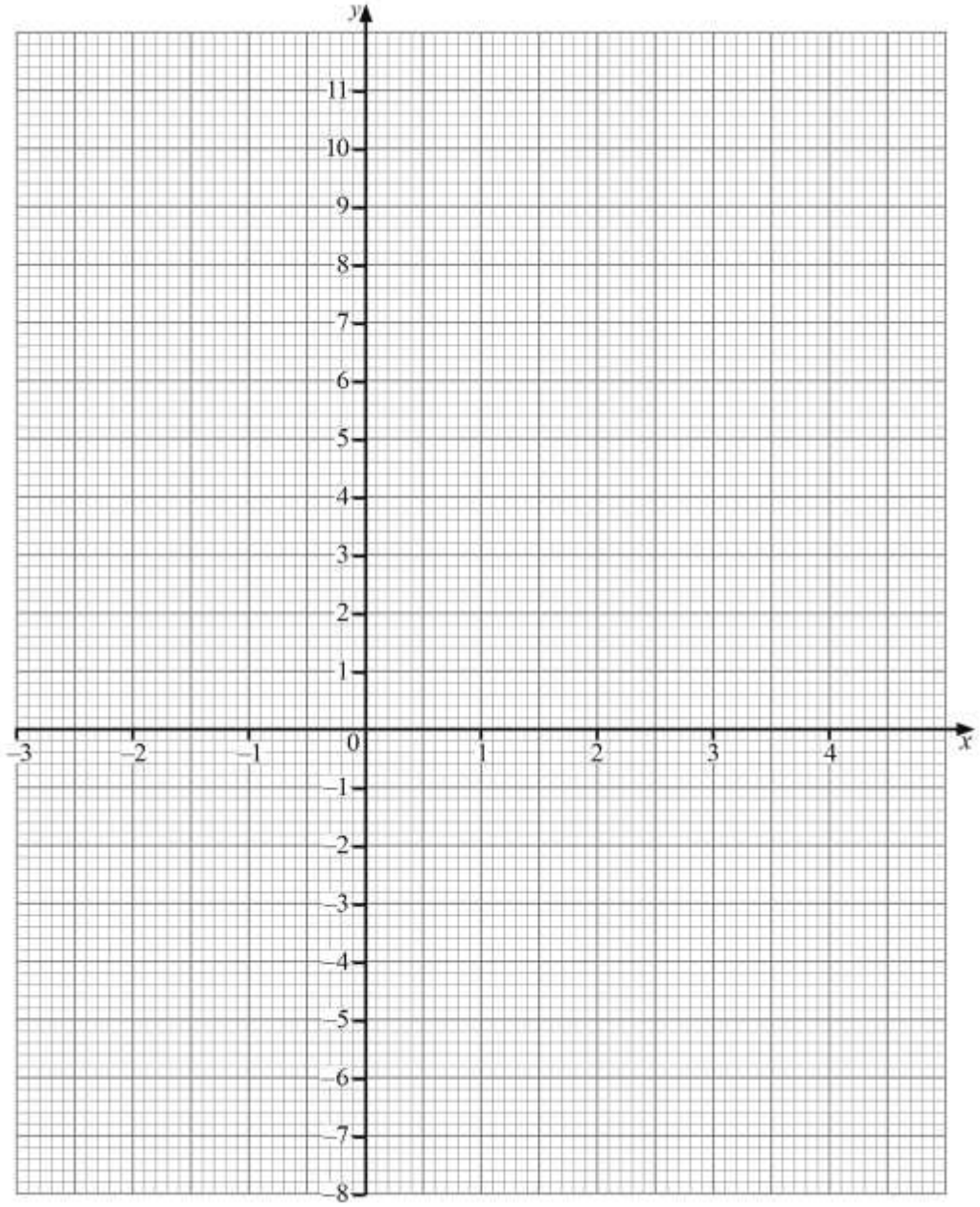
x	–2	–1	0	1	2	3	4
y		1	–5	–7		1	

[2]

(b) Draw the graph of $y = 2x^2 - 4x - 5$ for $x = -2$ to $x = 4$ on the opposite page. [2]

(c) Draw the line $y = -2$ and find the x values of the points of intersection.

Answer _____ [2]


y
x
11
10
9
8
7
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
−7
−8
−3
−2
−1
1
2
3
4

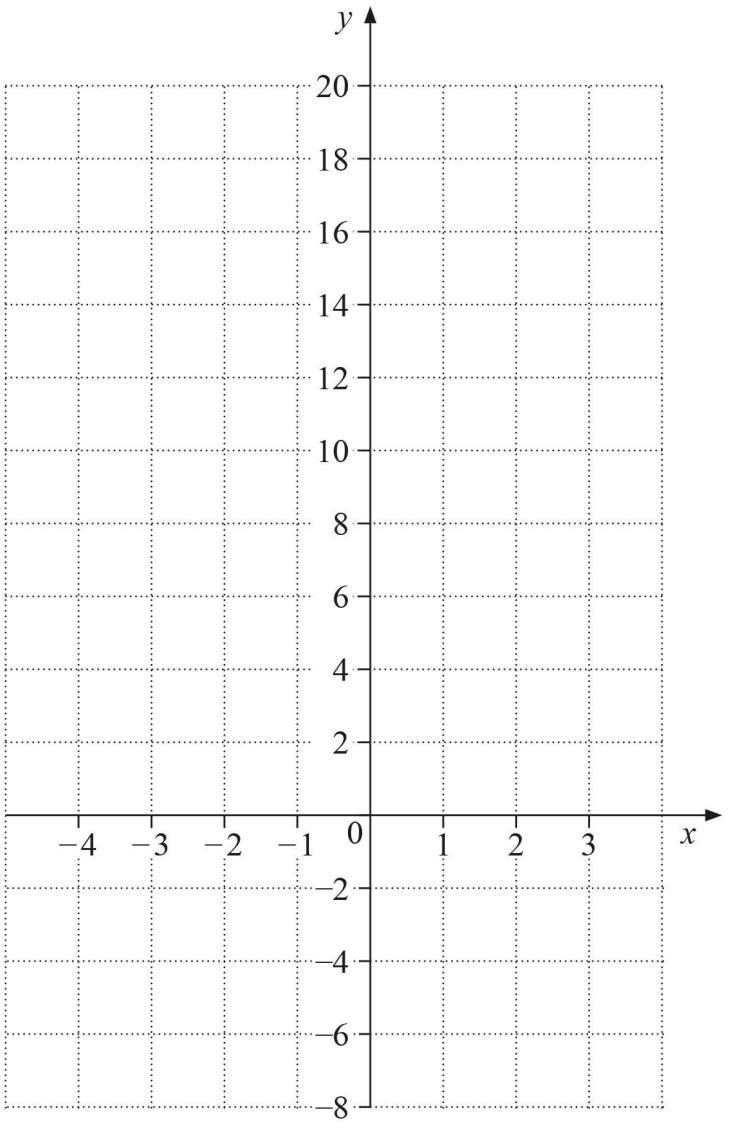
Q4

(a) Complete the table of values for $y = 3x^2 + 6x - 4$

x	-4	-3	-2	-1	0	1	2
y	20	5	-4		-4	5	20

[1]

(b) Hence, draw the graph of $y = 3x^2 + 6x - 4$ on the grid below.

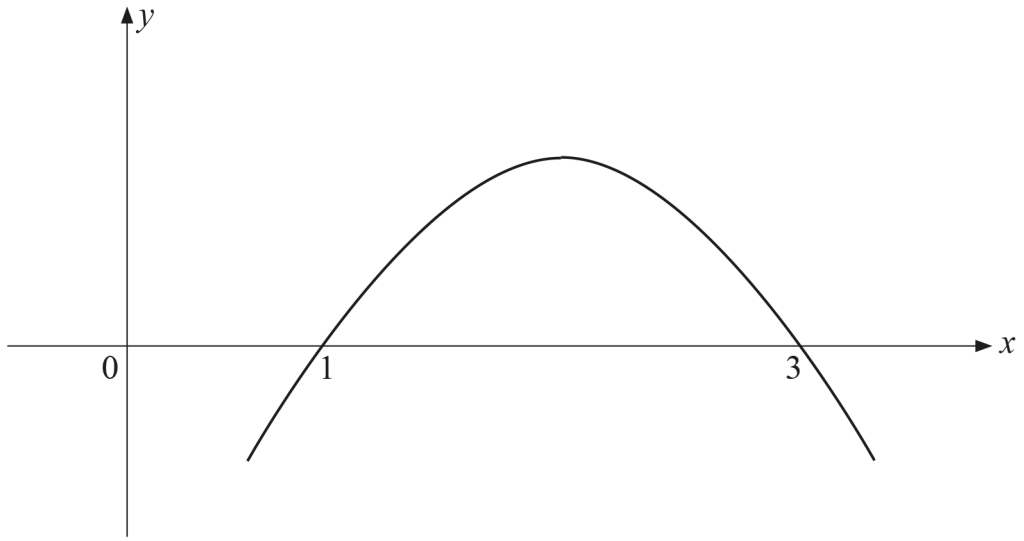


[2]

(c) Draw the line $y = 12$ on the grid.
Write down the x values of the points of intersection with this line.

Answer _____ , _____ [2]

Q5



The sketch above shows part of the graph of the quadratic function $y = -x^2 + 4x - 3$

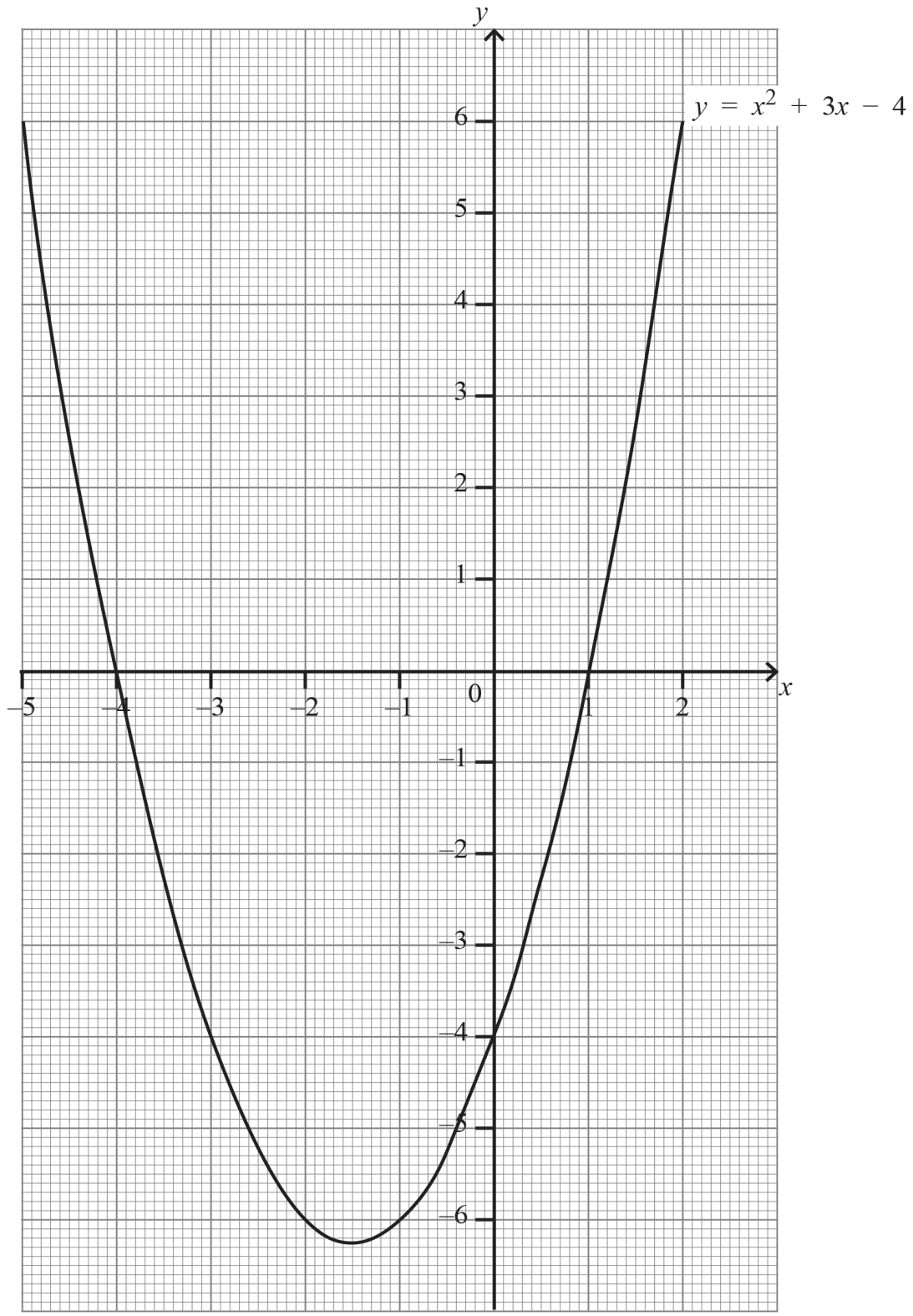
(a) Write down the coordinates of the point where the graph will cross the y-axis.

Answer _____ [1]

(b) Work out the coordinates of the highest point on the graph.

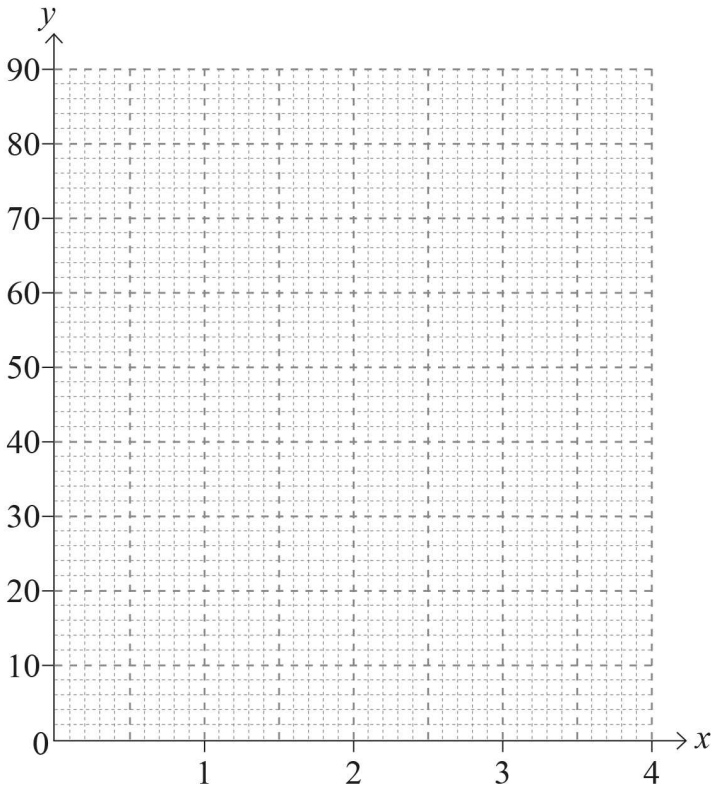
Answer _____ [1]

Q6



Q7

(a) Draw the graph of $y = 3^x$ for values of x between 0 and 4



[3]

(b) In the graph above, £y represents the value of one share in a company x years after purchase.

(i) What is the initial value of one share?

Answer £ _____ [1]

(ii) Describe what is happening to the value of the share each year.

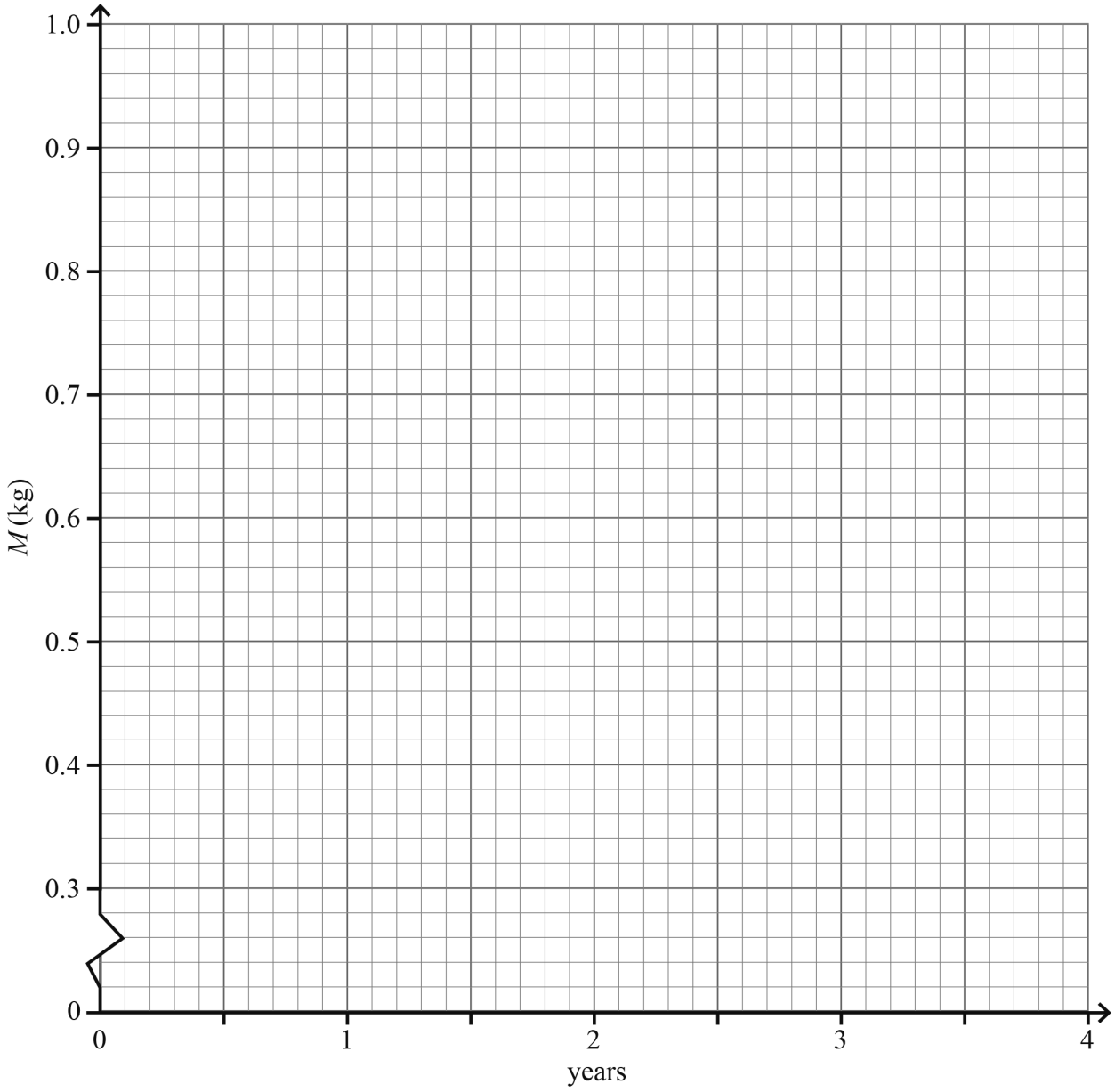
Answer _____ [1]

(c) By drawing a suitable line on the graph, estimate the instantaneous rate of increase in the value of the share when $x = 3$

Answer _____ £/year [2]

Q8 The mass Mkg of a decaying substance, x years after its manufacture, is given by the formula $M = (0.8)^x$

(a) Sketch a graph to show the mass over the first 4 years. [4]



(b) By drawing appropriate lines on the graph, find

(i) after how many months the mass was 70% of its original value,

Answer _____ months [2]

(ii) the rate of decay when $x = 3$

Answer _____ kg/year [2]

Q9

A container of radioactive waste, with an initial radioactivity of 100, is buried deep underground.

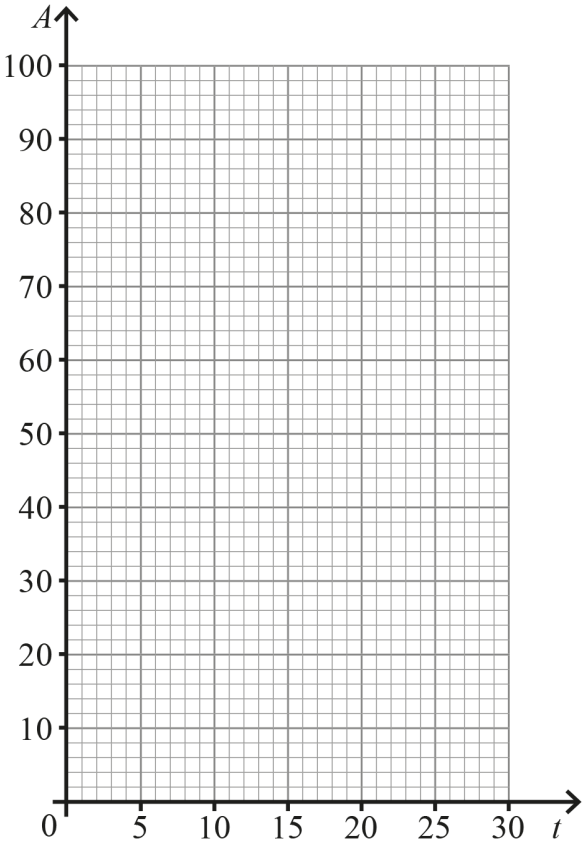
It decays by 5% each year.

After t years, its radioactivity A is given by the equation

$$A = 100 \times 0.95^t$$

(a) Complete the following table and use it to draw the graph of A against t on the grid below.

t	0	5	10	15	20	25	30
A							



[4]

(b) The half-life of a radioactive substance is defined as the time taken for the radioactivity of the material to decay to half its initial value.

Use your graph to estimate the expected half-life of this container of radioactive waste.

Answer _____ years [1]

1.

(a)	quadratic graph, quality	M1 A1
(b)	tangent drawn, appropriate value	M1 A1
(c)	$x^2 - 2x - \frac{1}{2} = 0$, so $x^2 - 2x + 3 = 3.5$	M1
	$y = 3.5$, x from graph	A1
(d)	$x^2 + 12x + 4 = 0$, so $x^2 - 2x + 3 = -14x - 1$	M1
	$y = -14x - 1$	A1

2.

(a)	A completely correct quadratic graph drawn (apply [–1] for each error, e.g. incomplete graph, or a wrong point, no proper minimum point, not a smooth curve, etc.)	A2
(b)	**(i)** $x = \frac{1}{2}$	A1
	(ii) $-2\frac{1}{4}$	A1

3.

(a)	11, –5, 11	A2
(b)	correct points smooth curve	A1 A1
(c)	–0.6 and 2.6 (follow candidate's graph)	A1 A1

4.

(a) -7 A1

(b) Points plotted correctly A1

Smooth curve A1

(c) Readings from graph A1 A1

5.

(a) $(0, -3)$ A1

(b) $(2, 1)$ A1

6.

(a) **(i)** Line $y = 2x - 1$ drawn MA1
Values from graph, around -2.3 and 1.3 MA1

(ii) $x^2 + x - 3 = 0$ MA1

(b) $x^2 + 3x - 4 = 3$ or $y = 3$ drawn MA1
Values from graph, around -4.5 and 1.5 A1

(c) $x^2 - x - 5 + 4x + 1 = 4x + 1$ or similar method MA1
$y = 4x + 1$ A1

7.

(a) using y values 1, 3, 9, 27, 81 MA2

Smooth sketch of curve A1

(b) **(i)** 1 A1

(ii) trebling A1

(c) tangent drawn at (3, 27) MA1

Values from drawn tangent A1

8.

(a) points (0,1), (1,0.8), (2,0.64), (3,0.512), (4,0.4096) (rounded)
Allow A2 for 4 points correct, A1 for 2 points correct
Smooth curve

(b) **(i)** reading from graph at 0.7, accurately

(ii) tangent drawn, read from graph

9.

(a)

t	0	5	10	15	20	25	30
A	100	77	60	46	36	28	21

A2

Points plotted correctly A1
Smooth curve through the points A1

(b) 13 to 14 A1
