St. Patrick's High School, Keady

Mathematics Department

GCSE Mathematics Practice Booklet

M8

Topic 6 – Geometry and Measure 2

Angles in Polygons

3D Pythagoras

Trigonometry (Sine rule, Cosine rule, ½ absinC)

Bearings, Constructions, Loci

Questions taken from CCEA Past Papers

Keady Maths

Q1

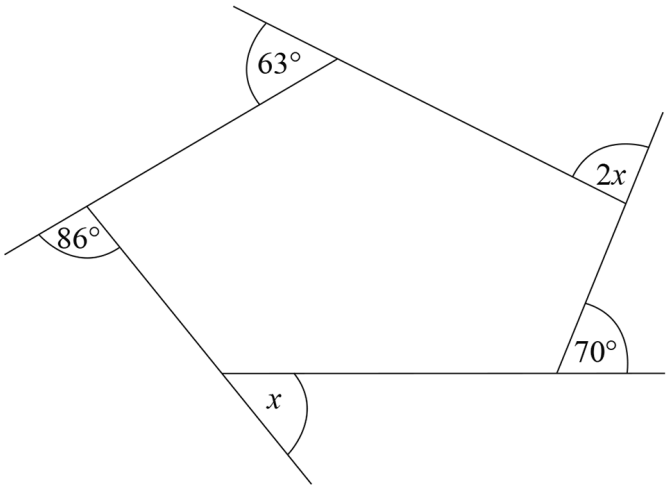


Diagram not drawn accurately

Work out the size of angle x in the diagram above.

Answer $x =$ _____ ° [4]

Q2

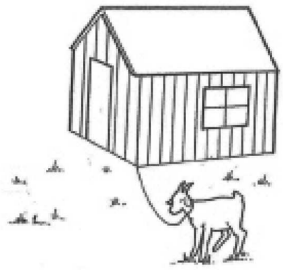
Each interior angle of a regular polygon is 140°.

How many sides has the polygon?

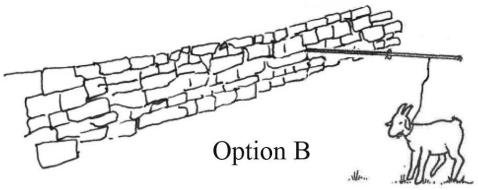
Answer _____ [2]

Q3 Farmer Jack wishes to tie his goat using a rope which is 4 m long.
He has three different options as illustrated below.
Option A: the rope is attached to a pole.
Option B: the rope is attached to (and can slide along and rotate around) a horizontal pole which extends 6 m at right angles from a very long wall.
Option C: the rope is attached to the corner of a shed which is 8 m long and 6 m wide.

Option A



Option C



Option B

Which option allows for the greatest grazing area for the goat?
What is the greatest area?
You must explain all your work clearly.
You may leave your calculations in terms of π where necessary.

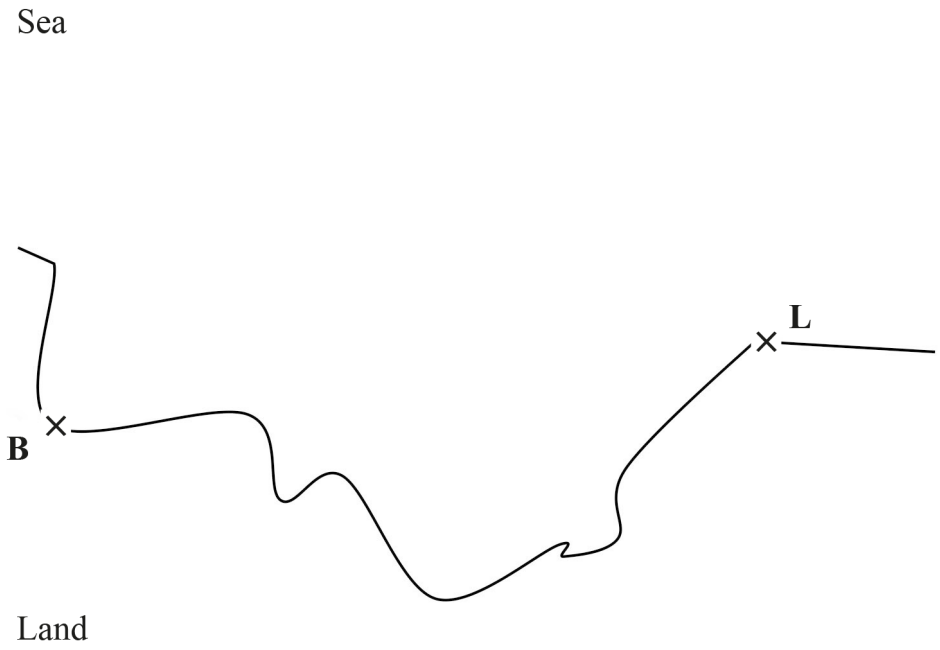
Answer Option _____ grazing area _____ m^2 [4]

Q4 The diagram shows a section of coastline with a lifeboat station marked at B and a lighthouse marked at L.

A sinking ship sends a distress signal.

The ship is less than 70 km from B and less than 30 km from L.

Using a scale of 1 cm = 10 km, **shade the region** in which the ship could be.



[3]

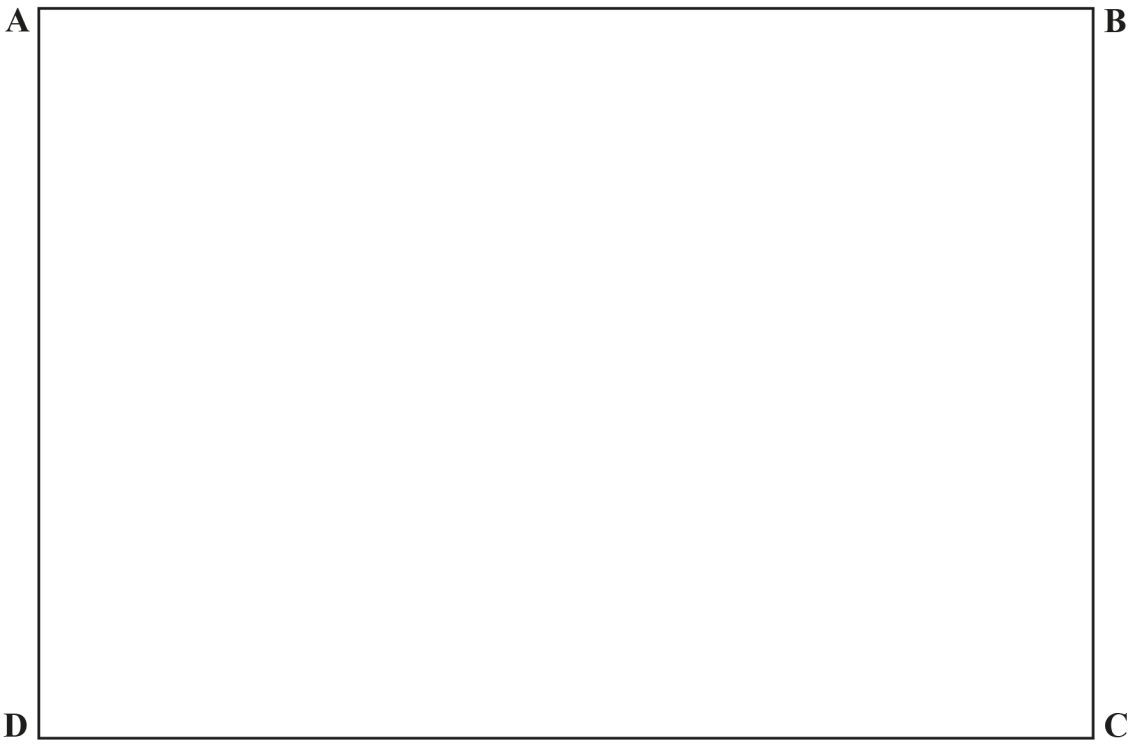
Q5 Toby walks his dog in the field **ABCD** so that he is always:

more than 40 m from **A**;

nearer to **A** than **B**;

nearer to **DA** than **DC**.

Shade the area where Toby walks his dog.



Scale of diagram: 1 cm = 10 m

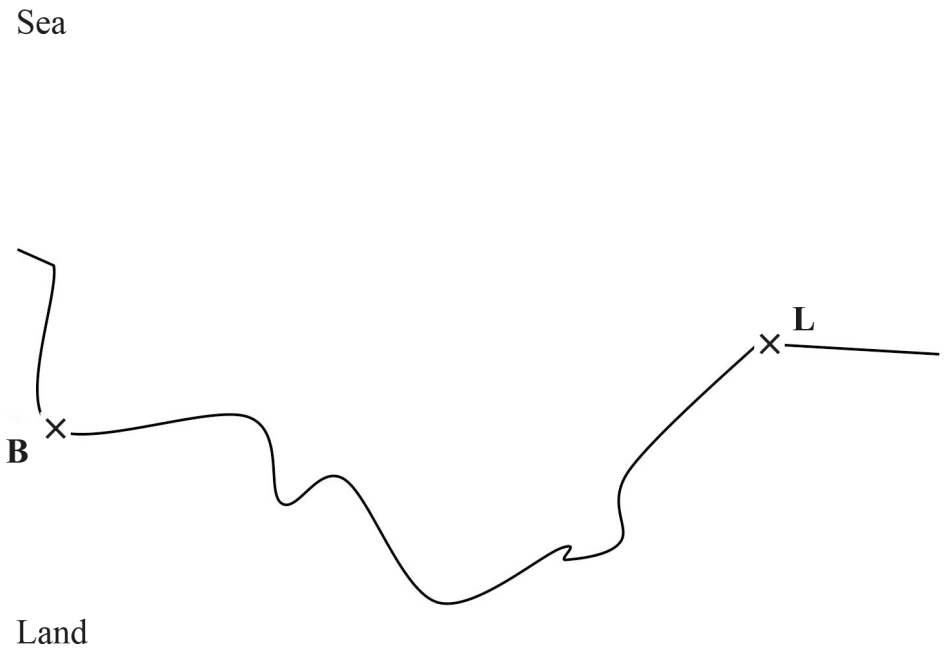
[4]

Q6 The diagram shows a section of coastline with a lifeboat station marked at B and a lighthouse marked at L.

A sinking ship sends a distress signal.

The ship is less than 70 km from B and less than 30 km from L.

Using a scale of 1 cm = 10 km, **shade the region** in which the ship could be.



[3]

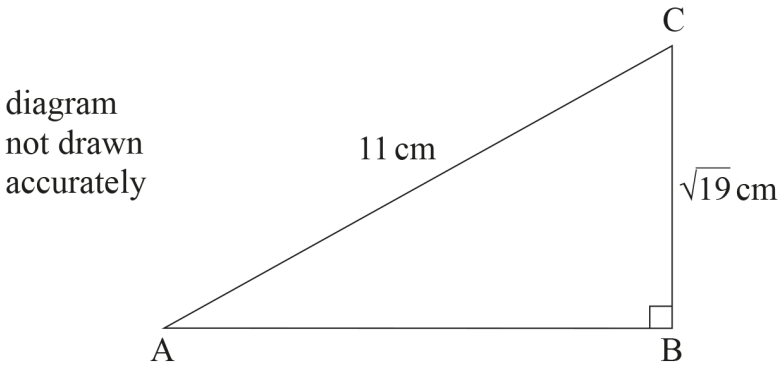
Q7

Use a ruler and compasses to construct the perpendicular from the point P to the line shown.

Leave all construction arcs and lines.

[2]

Q8



ABC is a right-angled triangle.

Work out the length of AB, giving your answer as a surd.

Answer _____ cm [3]

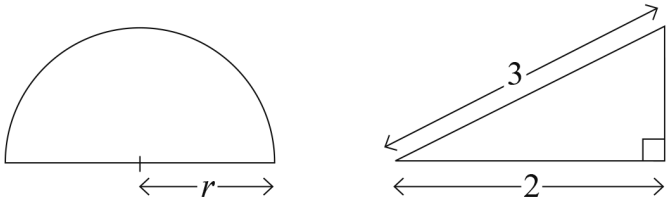
Q9 A cuboid has sides of length

$$3 + 2\sqrt{3}, \qquad 3 + \sqrt{3}, \qquad 9 - \sqrt{3}$$

Find the length of the space diagonal, giving your answer in the form $a\sqrt{b}$

Answer _____ [5]

Q10

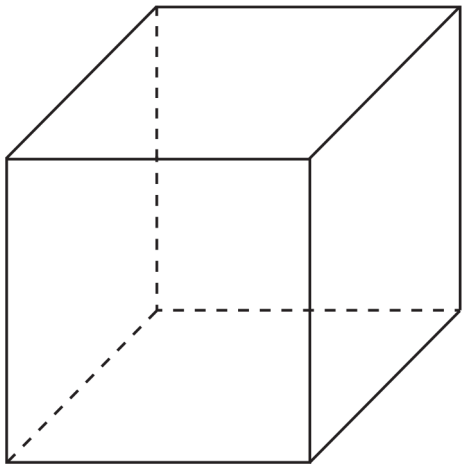


The semicircle and the triangle have the same perimeter.

Find the exact value of r, giving your answer in terms of π.

Answer r = _____ [4]

Q11



The length of the space diagonal of a cube is 9 cm.

Find the length of a side of the cube, giving your answer in the form $a\sqrt{b}$

Answer _____ cm [3]

1.

$x + 86 + 63 + 2x + 70 = 360$ or equivalent	M1 A1
$(3x =)\ 141$	MA1
$x = 47$	MA1

Alternative method:

$360 - (86 + 63 + 70)$ or $360 - 219$	M1 A1
141	MA1
47	A1

2.

$(180 - 140 =)\ 40$	MA1
$\left(\frac{360}{40} =\right) 9$	MA1

3.

Option A: circle area = 16π or 50.24	C1
Option B: $2 \times 4 \times 6 + \frac{1}{2} \times \pi \times 16 = 48 + 8\pi$ or 73.12	C1
Option C: $\frac{3}{4} \times \pi \times 16 = 12\pi$ or 37.68	C1
Option B with correct area of $48 + 8\pi$ or 73.12	A1

4.

Arc of 7 cm drawn from B	MA1
Arc of 3 cm drawn from L	MA1
Area bounded by the two arcs shaded	A1

5.

Arc, radius 4 cm, centre A	MA1
Bisector of AB	MA1
Bisector of angle D	MA1
Correct shading for arc and 2 lines	MA1

6.

Arc of 7 cm drawn from B	MA1
Arc of 3 cm drawn from L	MA1
Area bounded by the two arcs shaded	A1

7.

Arc from P crossing the line	MA1
Arcs from crossing point to intersect and draw line from P to intersection	MA1

8.

$AB^2 = 11^2 - (\sqrt{19})^2$	M1
$AB^2 = 102$	A1
$AB = \sqrt{102}$	A1

9.

$d^2 = (3 + 2\sqrt{3})^2 + (3 + \sqrt{3})^2 + (9 - \sqrt{3})^2$	MA1
$d^2 = 9 + 12\sqrt{3} + 12 + 9 + 6\sqrt{3} + 3 + 81 - 18\sqrt{3} + 3$	MA2
$d^2 = 117$	MA1
$d = 3\sqrt{13}$	A1

10.

Semicircle perimeter = $\pi r + 2r$	MA1
height = $\sqrt{9-4} = \sqrt{5}$	MA1
$\pi r + 2r = 2 + 3 + \sqrt{5}$	MA1
$r(\pi + 2) = 5 + \sqrt{5}$	
$r = \frac{5+\sqrt{5}}{\pi+2}$	A1

11.

$x^2 + x^2 + x^2 = 81$	MA1
$x^2 = 27$ so $x = \sqrt{27}$	MA1
$3\sqrt{3}$	A1

Q1 The diagram shows a regular pentagon placed on top of a regular hexagon.

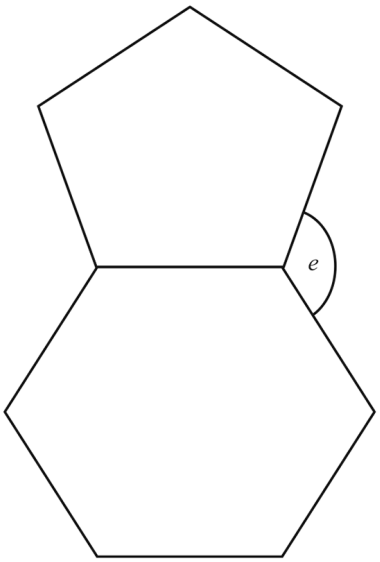


diagram not
drawn accurately

Calculate the size of the angle marked e.

Show all your working.

Answer e = _____ ° [4]

Q2 A regular polygon has an interior angle of 150°

(a) How many sides does it have?

Answer _____ [2]

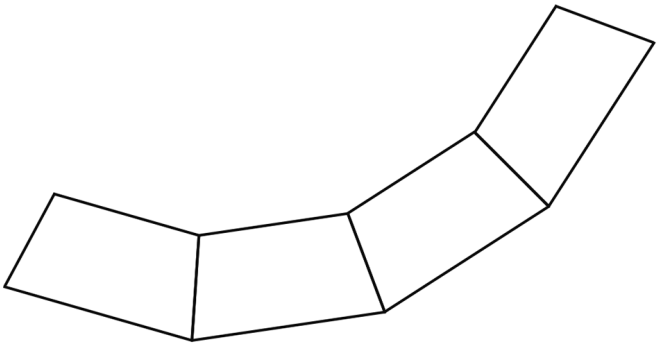
Two of these polygons are placed edge to edge.

(b) What regular shape would fit exactly in the space beside these touching edges?

Answer _____ [2]

Q3 The diagram shows a tile in the shape of an isosceles trapezium.

Some of these tiles are put together as a path all the way around a garden as shown.



How many exterior sides will the path have?

Show all your working clearly.

Answer _____ [3]

Q4

(a) Work out the size of an exterior angle of a 24-sided regular polygon.

Answer _____° [2]

(b) The sum of the interior angles of a regular polygon is 1800°

Work out how many sides this polygon has.

Answer _____ [2]

Q5 A regular polygon has exterior angles of size 15°

(a) How many sides has the polygon?

Answer _____ [2]

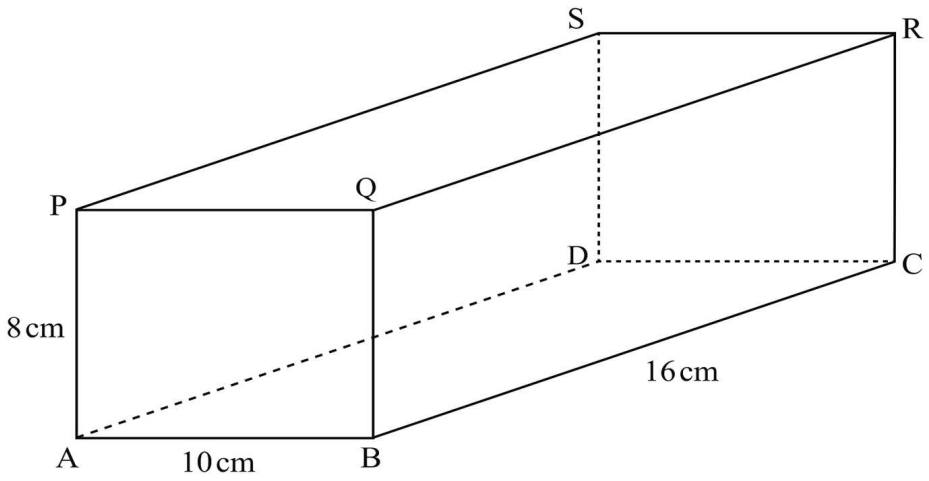
(b) Bailey thinks all regular pentagons are congruent.

Is he correct?

Circle your answer.

yes no more information needed [1]

Q6



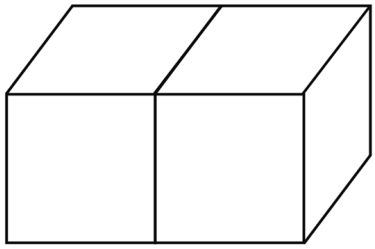
(a) Calculate the length of the space diagonal PC of the cuboid.

Answer _____ cm [2]

(b) Find the angle between PC and the face PSDA.

Answer _____ ° [3]

Q7

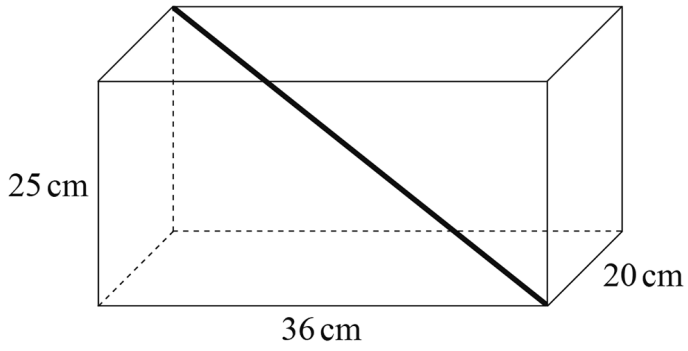


Two cubes, each of side 1 cm, are set side by side.

Calculate the angle between the base and the space diagonal from the bottom left hand corner to the top right hand corner.

Answer _____° [3]

Q8 The diagram below shows the position of the longest rod that can fit inside a box.



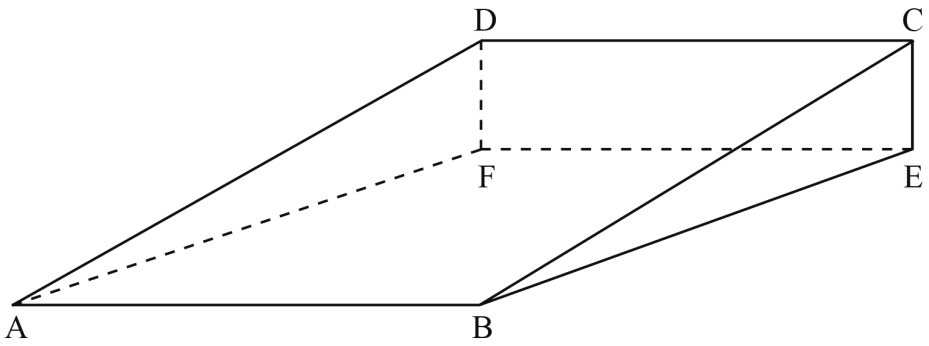
(a) Calculate the length of the rod.

Answer _____ cm [2]

(b) Calculate the angle that the rod makes with the base of the box.

Answer _____° [3]

Q9 **12**



The rectangular ramp ABCD is inclined to the horizontal rectangular base ABEF.

CDFE is vertical. AB = 6 m, CE = 4 m and BE = 9 m.

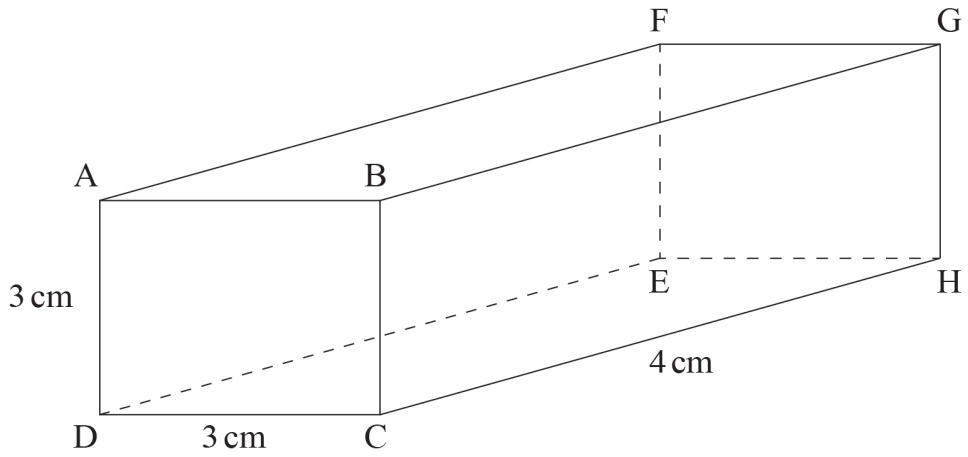
(a) Calculate the length of the line AC.

Answer _____ m [2]

(b) Calculate the angle between AC and the base.

Answer _____° [3]

Q10 ABCDEFGH is a cuboid with sides 3 cm, 3 cm and 4 cm as shown.



Calculate the angle between the space diagonal DG and the base DCHE.

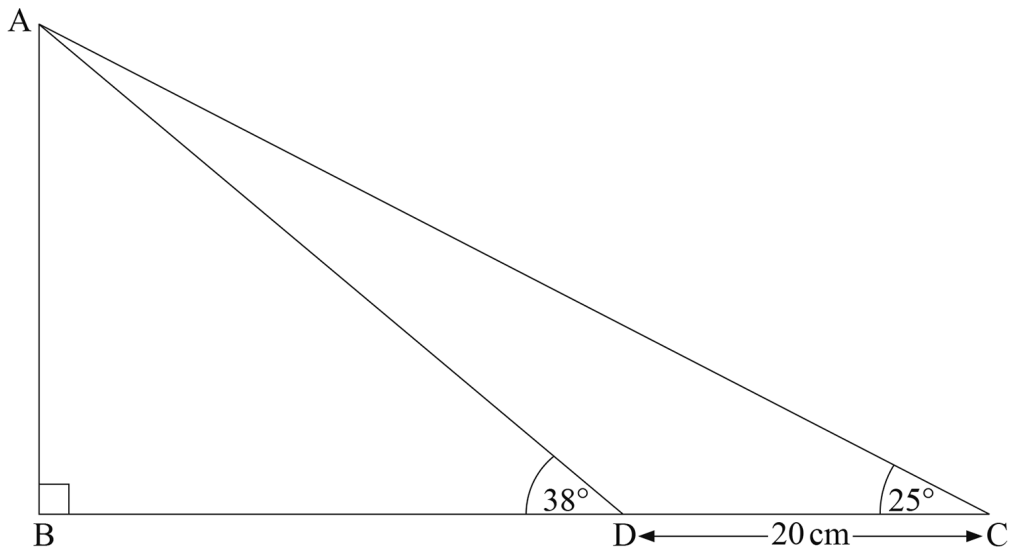
Answer _____ ° [3]

Q11 In the triangle shown

angle ABC = 90°, angle ADB = 38°, angle ACD = 25°

The length of DC = 20 cm.

Find the area of the triangle ABC.



Answer _____ cm^2 [5]

Q12 Two cars are travelling away from a crossroads on two straight roads.

At noon, one car is at A, 100 m from the crossroads, C, and the other is at B, 80 m from C. The distance AB is 82.3m. A short time later the first car has travelled 200m from A to D. The second car has travelled 250m from B to E.

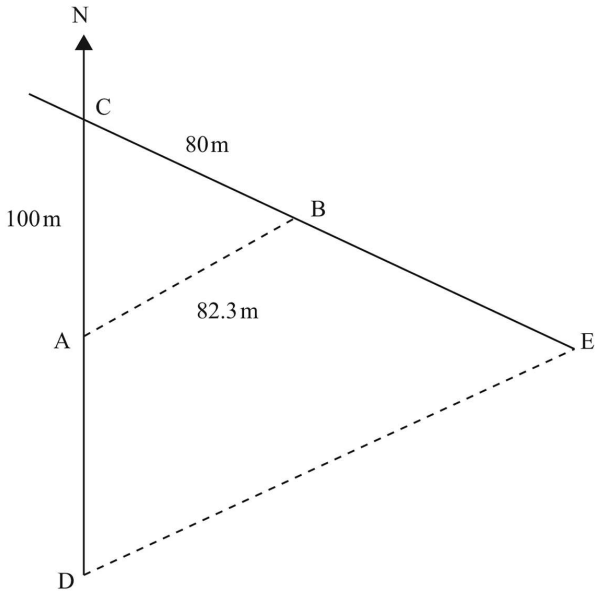


diagram not drawn accurately

(a) Find the distance, DE, between the two cars.

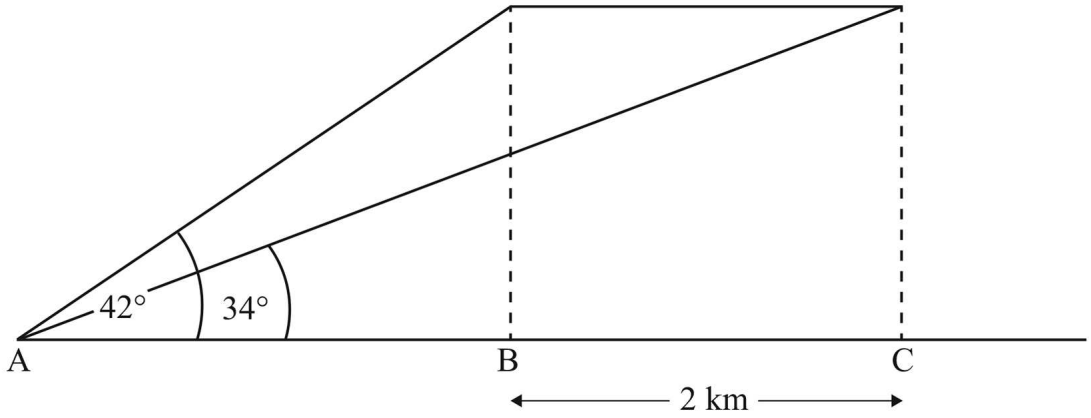
Answer _____ m [5]

(b) Find the bearing of E from D.

Answer _____ ° [3]

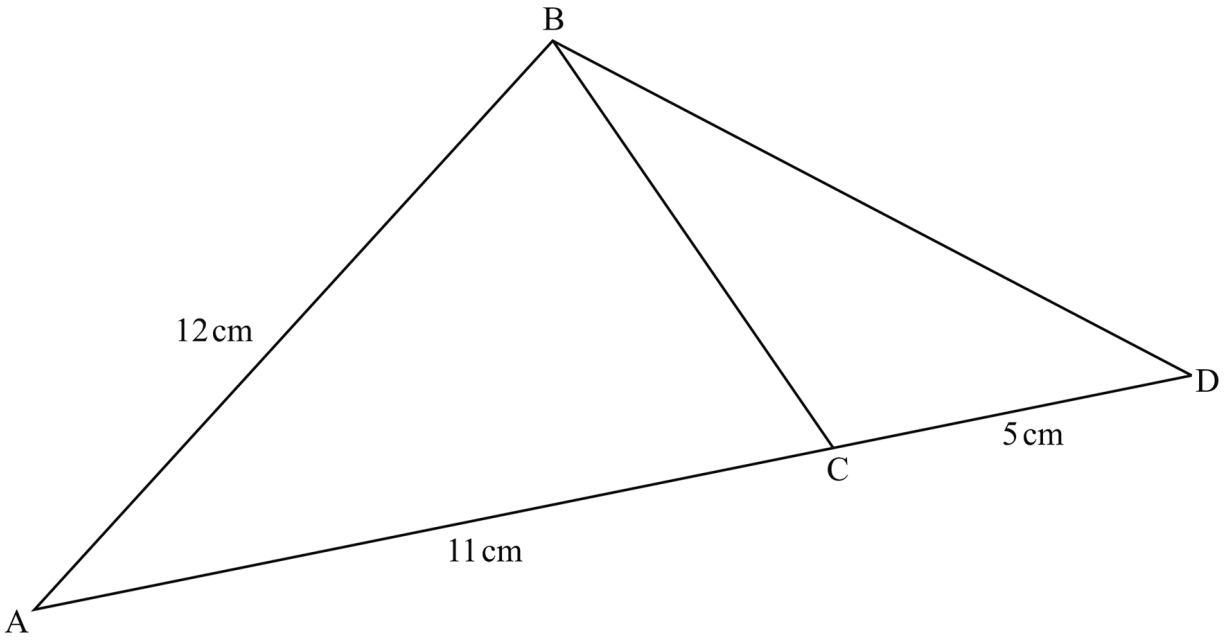
Q13 A, B, C are three points in a straight horizontal line.
A plane is flying horizontally directly above ABC.
When the plane passes over the point B, the angle of elevation from A is 42°.
When the plane passes over the point C, the angle of elevation from A is 34°.
The horizontal distance between B and C is 2 km.

Find the height of the plane above ABC to the nearest km.



Answer _____ km [6]

Q14 The area of the triangle ABC in the diagram below is $49\,cm^2$
The angle A is acute.
ACD is a straight line.

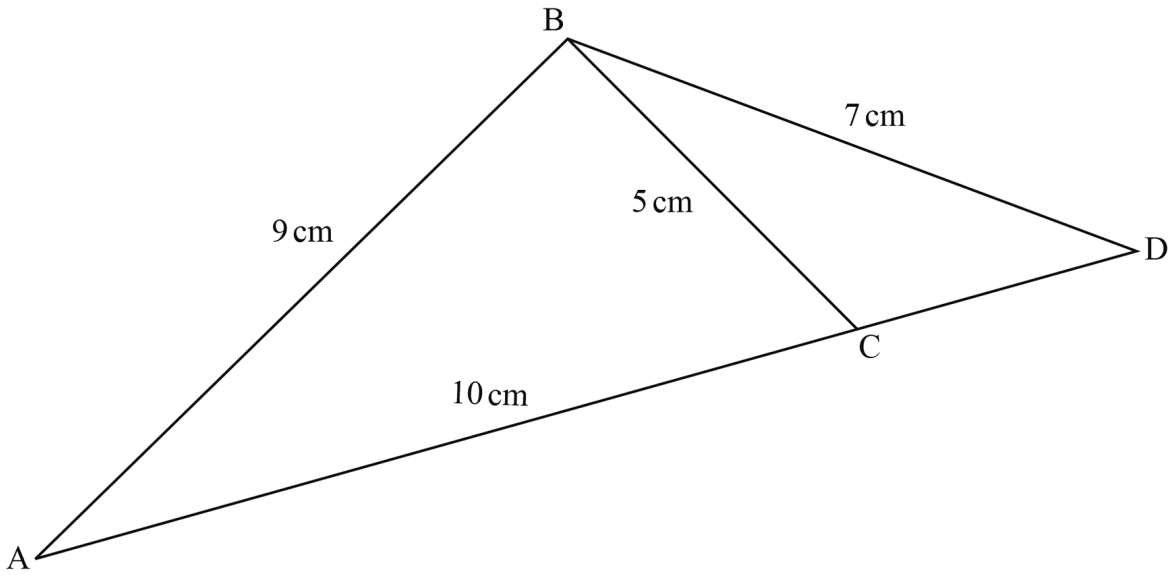


Find the area of the triangle BCD.

Answer _____ cm^2 [4]

Q15 The triangle ABC in the diagram shown has sides of 5, 9, 10 cm.

BD has length 7 cm. ACD is a straight line.



Calculate the size of the angle BDC.

Answer _____° [5]

Q16

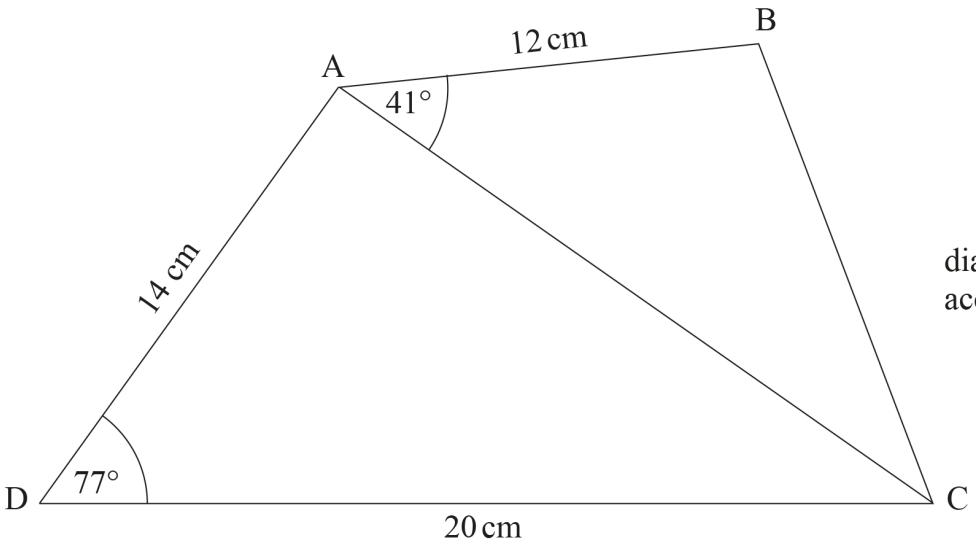
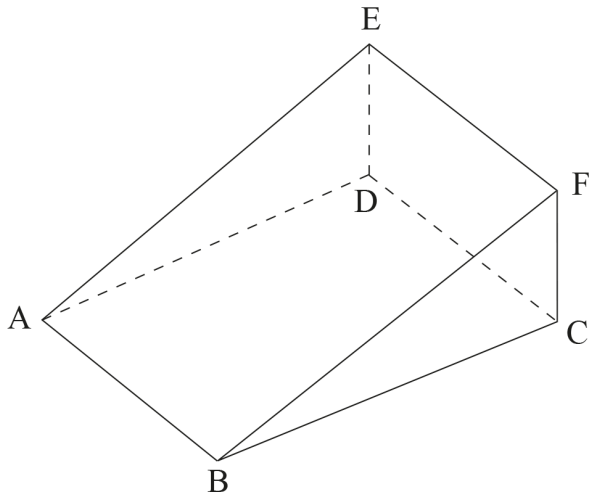


diagram not drawn accurately

Find the area of the triangle ABC.

Answer _____ cm^2 [5]

Q17



ABCDEF is a triangular prism with ABCD a horizontal rectangle and CDEF a vertical rectangle.

AB = 20 cm, BC = 28 cm and CF = 14 cm.

Calculate the difference in size of the angles of elevation EAD and EBD.

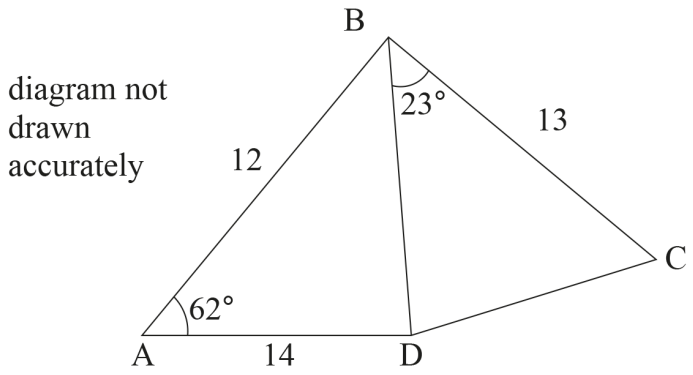
Answer _____° [6]

Q18 The lengths of the sides of a triangle are 6 cm, 7 cm and 8 cm.

Calculate the three angles in the triangle.

Answer _____° , _____° , _____° [6]

Q19

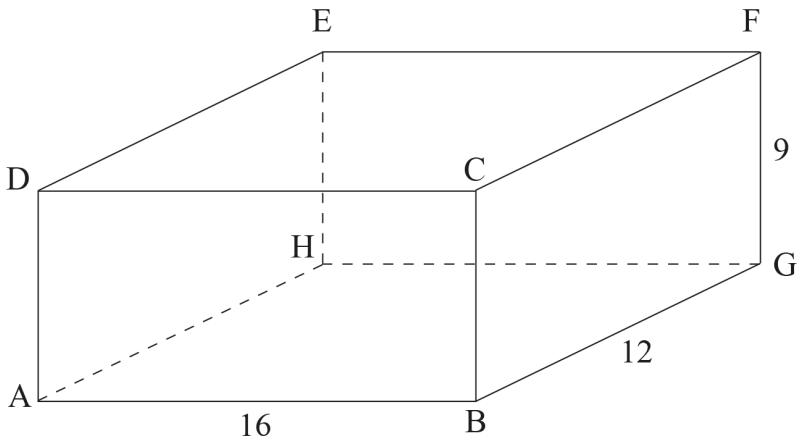


AB = 12 cm, AD = 14 cm and BC = 13 cm.

Calculate the area of the triangle BCD.

Answer _____ cm^2 [5]

Q20



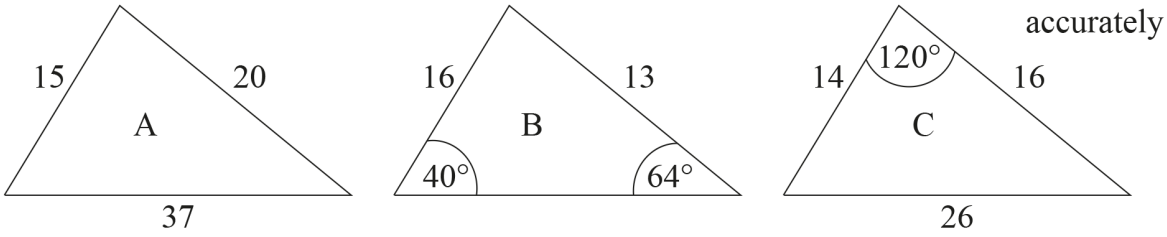
ABCDEFGH is a cuboid with sides 9 cm, 12 cm and 16 cm as shown.

Calculate the size of the angle AEC.

Answer _____° [6]

Q21 Explain clearly which of the diagrams below can represent triangles.

diagrams not drawn accurately



[5]

Q22

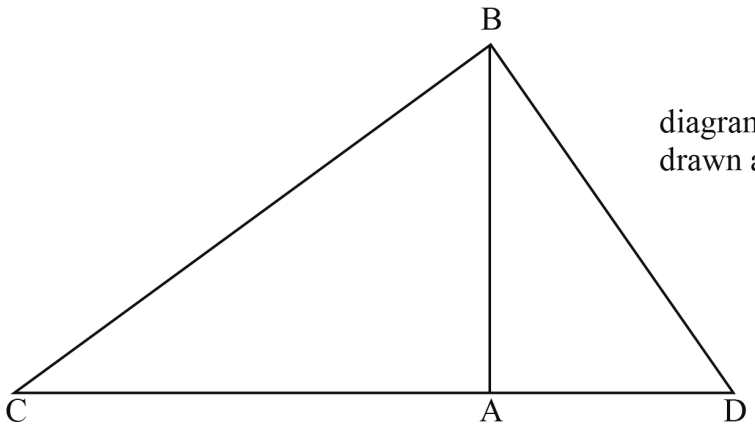


diagram not
drawn accurately

AB is a vertical mast and CAD is horizontal.

The angle of elevation of B from C is 36˚

The angle of elevation of B from D is 48˚

The distance CD is 100 metres.

Calculate the height of AB.

Answer _____ m [5]

Q23

B is 30 km East and 20 km North of A.
C is 40 km from B and 60 km from A.

Calculate the bearing of C from B.

A solution by scale drawing will not be accepted.

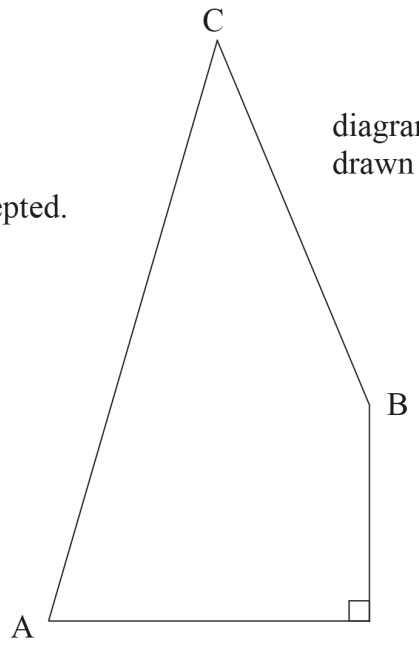


diagram not drawn accurately

Answer _____° [7]

Q24

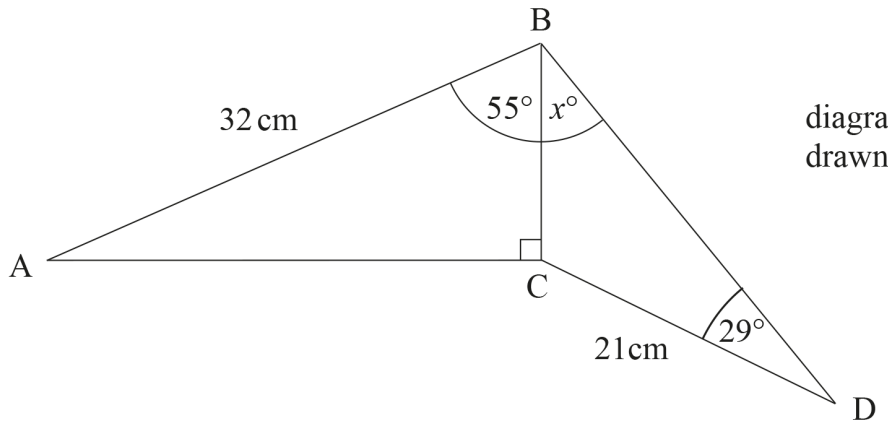


diagram not drawn accurately

AB = 32 cm and angle ABC = 55°

CD = 21 cm and angle BDC = 29°

Calculate the size of angle CBD.

Answer _____ ° [5]

Q25

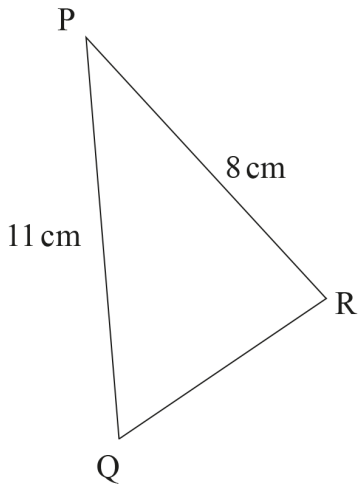


diagram not drawn accurately

In the triangle PQR, the angle P is acute, PQ = 11 cm and PR = 8 cm

The area of PQR is 22 cm^2

Calculate the length of QR.

Answer _____ cm [5]

Q26

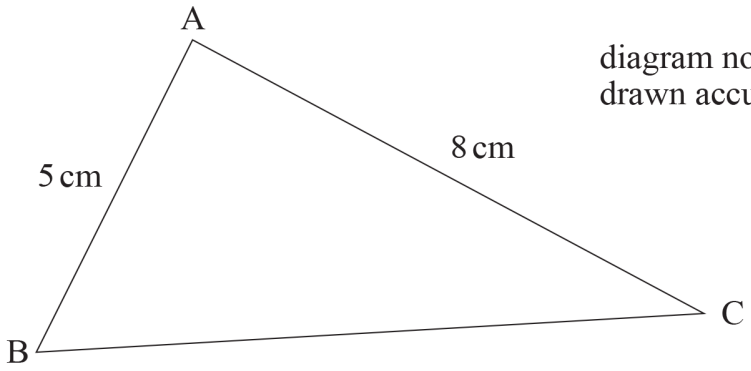


diagram not
drawn accurately

The area of ABC is 16 cm^2

Find the length of BC.

Answer _____ cm [5]

1.

$360 \div 5 = 72$ **or** $360 \div 5 = 72$		**or** $540 \div 5 = 108$	MA1
$360 \div 6 = 60$	$360 \div 6 = 60$	$720 \div 6 = 120$	MA1
$72 + 60$	$108 + 120$	$108 + 120 = 228$	MA1
132	$360 - (108 + 120) = 132$	$360 - 228 = 132$	MA1

2.

(a) Exterior angle = 30 MA1

Number of sides = $\frac{360}{30} = 12$ MA1

(b) equilateral triangle

Allow A1 for sight of 60° in calculation or diagram A2

3.

$180 - 102 = 78$

$78 \times 2 = 156$ C1

$180 - 156 = 24$ C1

$360 \div 24 = 15$ sides C1

4.

(a) $360 \div 24$ M1

15 A1

(b) $180\ (n - 2) = 1800$ M1

$n = 12$ A1

5.

(a) $360 \div 15 = 24$ M1A1

(b) no A1

6.

(a) $\sqrt{(10^2 + 16^2 + 8^2)} = \sqrt{420}$ MA1

$= 20.5$ A1

(b) Angle CPD required M1

$\sin x = \frac{10}{\sqrt{420}}$ or $\cos x = \frac{\sqrt{320}}{\sqrt{420}}$ or $\tan x = \frac{10}{\sqrt{320}}$ MA1

$= 29.2°$ A1

7.

diagonal $= \sqrt{6}$ MA1

$\sin x = \frac{1}{\sqrt{6}}$ **or** $\tan x = \frac{1}{\sqrt{5}}$ **or** $\cos x = \frac{\sqrt{5}}{\sqrt{6}}$ M1

$x = 24.09484255$ rounded A1

8.

(a) $\sqrt{(36^2 + 20^2 + 25^2)}$ M1

$= 48.17675\ldots\ldots$ A1

Alternative method

$\sqrt{(36^2 + 20^2)} = 41.18252056$ MA1

$\sqrt{(41.18252056^2 + 25^2)} = 48.17675\ldots$ MA1

(b) Diagonal of base $= 41.18252056$ MA1

$\text{Tan } \alpha = \frac{25}{41.18252056}$ MA1

$\alpha = 31.3°$

9.

(a) $AC^2 = 6^2 + 4^2 + 9^2$ $(= 133)$ MA1

$AC = 11.5\ (3256259\ldots)$ A1

(b) using $\angle$ CAE M1

$\sin \angle CAE = \frac{4}{11.5(3256259\ldots)}$ M1

$\angle CAE = 20.2944\ \ldots$ A1

10.

Use of angle GDH MA1

$\tan GDH = \frac{3}{5}$ or $\sin GDH = \frac{3}{\sqrt{(9 + 9 + 16)}}$ or $\cos GDH = \frac{5}{\sqrt{(9 + 9 + 16)}}$ MA1

$30.96(375653\ldots)°$ A1

11.

Angle DAC = 13 °	A1
$\frac{20}{\sin 13} = \frac{AD}{\sin 25}$	
$AD = \frac{20 \sin 25}{\sin 13} = 37.57\ (424147\ldots)$	MA1
BD = 37.57 cos 38 ° = 29.61	MA1
AB = 37.57 sin 38° = 23.13	MA1
Area = 0.5 × 49.61 × 23.13 = 573.7	MA1

12.

(a)	$82.3^2 = 80^2 + 100^2 - 2(80)(100) \cos C$	MA1
	$\cos C = \frac{100^2 + 80^2 - 82.3^2}{2(100)(80)}$	MA1
	$x = 53°$	MA1
	$DE^2 = 330^2 + 300^2 - 2 \times 330 \times 300 \times \cos 53$	MA1
	DE = 282.4	A1
(b)	$\frac{330}{\sin a} = \frac{282.4}{\sin 53}$	MA2
	a = 68.9° bearing 068.9°	MA1

13.

Top angle is 180 – (8 + 34) = 138 or 34°	MA1
2/sin 8 = x/sin 138	MA2
x = 9.616 km	A1
Height = 9.616 sin 34	MA1
= 5.377 km = 5 km	A1

14.

$49 = \frac{1}{2}(12)(11)\sin A$	**or**	$\frac{1}{2} \times 11 \times h = 49$	MA1
$\sin A = \frac{49}{66}$			
$A = 47.94°$		$h = 8.90(90 \ldots)$	A1
Area ABD $= \frac{1}{2}(12)(16)\ \sin 47.94° = 71.27$		$\frac{1}{2} \times 5 \times 8.90$	MA1
So Area BCD $= 71.27 - 49$			
$= 22.27\,\text{cm}^2$		$= \quad 22.27\ \text{cm}^2$	A1

15.

$\cos \text{ACB} = \frac{(100 + 25 - 81)}{100} = 0.44$	MA2
$\text{ACB} = 63.89611886 \quad (\text{DCB} = 116.10388114)$	MA1
$\sin \text{BDC} = \frac{5 \sin \text{DCB}}{7} = 0.6414269806$	MA1
Ans 39.89830833 (rounded)	A1

16.

$d^2 = 14^2 + 20^2 - 2(14)(20)\cos 77°$	M1 A1
$d = 21.68011553$	A1
$\text{Area} = \frac{1}{2}\ (12)\ (21.68011553)\sin 41° = 85.3(4061327)$	M1 A1

17.

$\tan EAD = \frac{14}{28}$ MA1

$EAD = 26.56505°$ A1

$DB^2 = 20^2 + 28^2$ so $DB = 34.4093$ MA1

$EBD = \tan^{-1}\left(\frac{14}{34.4093}\right) = 22.13977°$ M1 A1

Difference $= 4.43°$ A1

18.

cos rule for one angle

$\cos = \frac{6^2 + 7^2 - 8^2}{2 \times 8 \times 7}$ angle = 75.52 M1 A1 A1

or $\cos = \frac{6^2 + 8^2 - 7^2}{2 \times 6 \times 8}$ angle = 57.91 M1 A1 A1

or $\cos \frac{7^2 + 8^2 - 6^2}{2 \times 7 \times 8}$ angle = 46.57 M1 A1 A1

sin rule for second angle $\frac{\sin 75.52}{8} = \frac{\sin ?}{6}$ or $\frac{\sin ?}{7}$? = 46.57 or 57.91 M1 A1

or $\frac{\sin 57.91}{7} = \frac{\sin ?}{6}$ or $\frac{\sin ?}{8}$? = 46.57 or 75.52 M1 A1

or $\frac{\sin 46.57}{6} = \frac{\sin ?}{7}$ or $\frac{\sin ?}{8}$? = 57.91 or 75.52 M1 A1

3rd angle correct MA1

Follow through all parts for numerical errors

19.

$BD^2 = 12^2 + 14^2 - 2 \times 12 \times 14 \cos 62° = 182.2575549$ M1 A1

$BD = 13.5$ A1

$\text{Area} = \frac{1}{2} \times 13.5 \times 13 \times \sin 23° = 34.29$ M1 A1

20.

use of triangle ACE	M1
$AC^2 = 9^2 + 16^2 = 337$	MA1
$AE^2 = 9^2 + 12^2 = 225$	MA1
$EC^2 = 12^2 + 16^2 = 400$	MA1
$\cos AEC = \frac{225 + 400 - 337}{2 \times 15 \times 20} = \frac{288}{600} = 0.48$	
61.3°	M1 A1

21.

$15 + 20 < 37$ not a triangle	MA1
$\sin 64°/16 \neq \sin 40°/13$ not a triangle	M1 A1
cos rule works for 14, 16, 26, 120° (shown) triangle	M1 A1

22.

$\frac{BC}{\sin 48°} = \frac{100}{\sin 96°}$ **or** $\frac{BD}{\sin 36°} = \frac{100}{\sin 96°}$ M1A1

BC = 74.7238….. **or** BD = 59.10229.... MA1

AB = BCsin36° **or** AB = BD sin48° MA1

AB = 43.92156…. A1

Alternative

$\tan 36° = \frac{h}{100 - x}$ and $\tan 48° = \frac{h}{x}$ M1A1

$(100 - x)\tan 36° = x \tan 48°$ MA1

$x = 39.547$ …….. MA1

$h = 43.92156$ …….. A1

23.

$AB^2 = 20^2 + 30^2 = 1300$ so AB = 36.055 M1 A1

$\cos B = \frac{(1300 + 1600 - 3600)}{(20 \times 40 \times 36.055)}$ M1 A1

= –0.242681 so B = 104.045 A1

$\tan^{-1} \frac{30}{20} = 56.3099$ MA1

Bearing = 180 + 56.3099 + 104.044 = 340(.35478) MA1

24.

$BC = 32 \cos 55° = 18.35444596$	M1 A1
$\frac{\sin x}{21} = \frac{\sin 29°}{18.35444596}$	M1 A1
$x = 33.68926935$	A1

25.

$\frac{1}{2} \times 11 \times 8 \times \sin P = 22$	MA1
$\sin P = 0.5, \quad P = 30°$	A1
$p^2 = 11^2 + 8^2 - 2 \times 11 \times 8 \times \cos 30°$	M1 A1
$p^2 = 32.57952893, \ p = 5.7(07848013)$	A1

26.

$\frac{1}{2} \times 5 \times 8 \sin A = 16$	MA1
$\sin A = 0.8$ so $A = 53.13010235$	A1
$a^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \cos (53.13010235) = 41$	M1A1
6.4(03124237)	A1
