Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2022

Further Mathematics

Unit 3 **(With calculator)**
Statistics

[GFM31]

GFM31

FRIDAY 24 JUNE, AFTERNOON

TIME

1 hour.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page.
Complete in black ink only. **Do not write with a gel pen.**
All working **must** be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all seven** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 50.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on page 2 and the Normal Probability Table is on page 3.

FORMULA SHEET

STATISTICS

Statistical measures: $\text{Mean} = \dfrac{\Sigma fx}{\Sigma f}$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$$

where $\bar{x}$ is the mean

Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

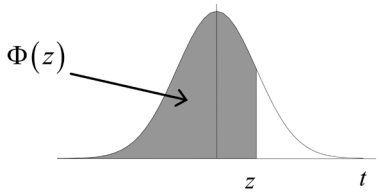
Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$$r = 1 - \frac{6 \Sigma d^2}{n(n^2 - 1)}$$

NORMAL PROBABILITY TABLE

Table of $\Phi(z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990



[Turn over

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1 The table below shows the numbers of pupils in a class who are right-handed and left-handed.

	Right-handed	**Left-handed**
Boys	8	3
Girls	5	7

→ 11 Boys

→ 12 Girls

One boy and one girl are selected at random.

Calculate the probability that both pupils are left-handed.

P(Boy Left Handed) × P(Girl left handed)

$\frac{3}{11} \times \frac{7}{12}$

$= \frac{21}{132} = \frac{7}{44}$

Answer $\frac{7}{44}$ [2]

[Turn over

2 Ten students had part-time jobs during the summer.

Jack recorded the numbers of hours they worked each week and the corresponding amounts they were paid each week. The results are shown in the table below.

Student	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Hours worked	40	24	15	32	10	48	16	28	12	35
Amount paid (£)	460	320	120	360	180	430	260	350	180	340
Ranks (Hours worked)	9	5	3	7	1	10	4	6	2	8
Ranks (Amount paid)	10	5	1	8	2.5	9	4	7	2.5	6
d	−1	0	2	−1	−1.5	1	0	−1	−0.5	2
d^2	1	0	4	1	2.25	1	0	1	0.25	4

(i) Write down, in the table above, the rank orders for the Hours worked and the Amount paid. [2]

(ii) Calculate Spearman's coefficient of rank correlation.

$$1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6(14.5)}{10(99)}$$

$$= 0.912121$$

Answer 0.91 [4]

(iii) Interpret your answer to part **(ii)**.

Answer Positive Correlation [1]

(iv) Calculate the mean Hours worked and the mean Amount paid.

TOTAL HOURS = 260 mean = 26

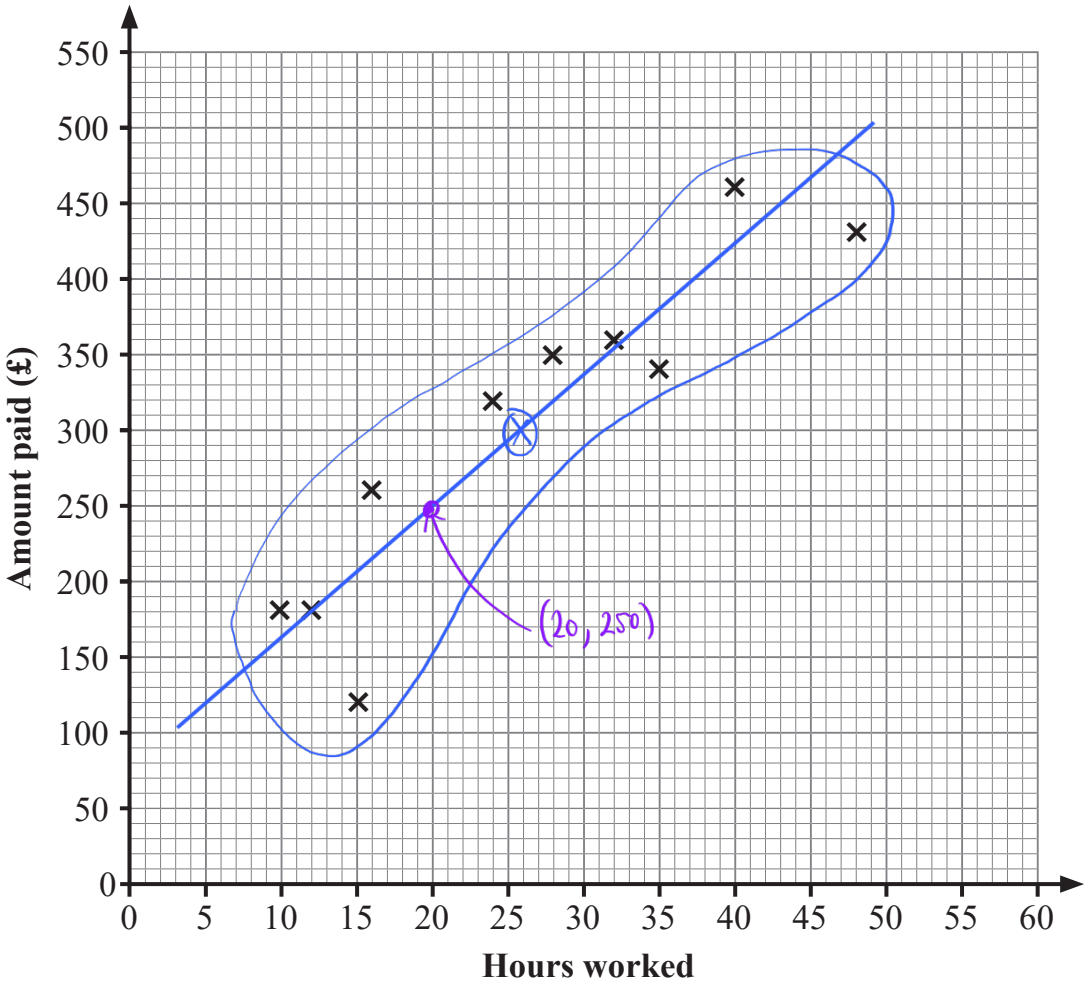
TOTAL AMOUNT = 3000 mean = 300

Answer Mean Hours worked 26 hours

Mean Amount paid £ 300 [1]

Q2 continues on page 8

The data from the table are plotted on the graph below.



(v) Draw your line of best fit on the graph. [2]

(vi) Determine the equation of the line of best fit which you have drawn.

$(x, y) \rightarrow (20, 250)$

$(\bar{x}, \bar{y}) \rightarrow (26, 300)$

$$m = \frac{\bar{y} - y}{\bar{x} - x} = \frac{300 - 250}{26 - 20}$$

$$= \frac{50}{6} = 8.333$$

$$y - \bar{y} = m(x - \bar{x})$$

$$y - 300 = 8.333(x - 26)$$

$$y - 300 = 8.333x - 216.67$$

$$y = 8.333x + 83.333$$

Answer $y = 8.33x + 83.3$ [3]

3 Paul recorded his golf scores over the course of a year.

His results are summarised in the table below.

Score	Frequency	x midpoint	fx freq × midpoint	fx²
65 – 67	4	66	264	17424
68 – 70	12	69	828	57132
71 – 73	24	72	1728	124416
74 – 76	32	75	2400	180000
77 – 79	6	78	468	36504
80 – 82	2	81	162	13122
	80		5850	428598

(i) Calculate an estimate of the mean score.

You **must show clearly** each step of your work using the table above.

$$\frac{\Sigma fx}{\Sigma f} = \frac{5850}{80} = 73.125$$

Answer 73.1 [2]

(ii) Explain why the answer for the mean in part **(i)** is only an estimate.

Answer midpoints used in place of actual values [1]

(iii) Calculate an estimate of the standard deviation of the scores.

You **must show clearly** each step of your work using the table opposite.

$$\text{s.d.} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$$

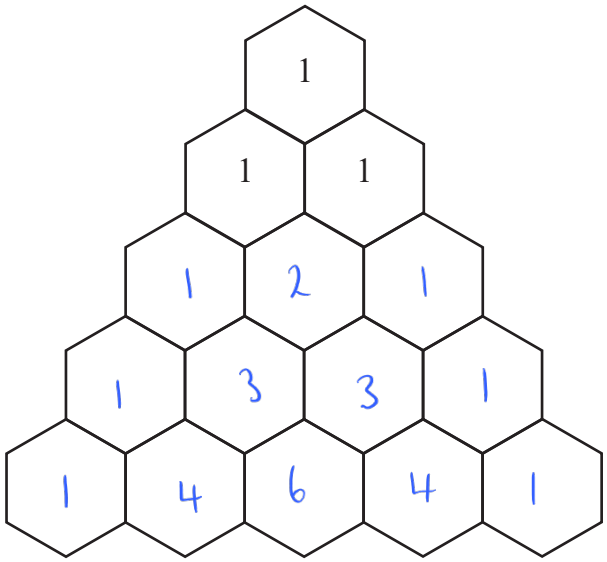
$$= \sqrt{\frac{428598}{80} - (73.125)^2}$$

$$= 3.195211261$$

Answer 3.20 [3]

[Turn over

4 **(i)** Complete Pascal's triangle in the grid below.



[1]

(ii) Hence write down the expansion of $(p + q)^4$

Answer $p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$ [1]

Charlotte goes into a room where there are four windows.

Each window is either open or closed.

The probability that a window is open is 0.7

$p = 0.7 \rightarrow$ open

$q = 0.3 \rightarrow$ closed

(iii) Find the probability that exactly three windows are closed.

$p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$

1 open, 3 closed

$4(0.7)(0.3)^3 = 0.0756$

Answer 0.0756 [3]

Q4 continues on page 14

(iv) Find the probability that at least two windows are closed.

$p^4 + 4p^3q + \boxed{6p^2q^2 + 4pq^3 + q^4}$

$= 6(0.7)^2(0.3)^2 + 4(0.7)(0.3)^3 + (0.3)^4$

$= 0.2646 + 0.0756 + 0.0081$

$= 0.3483$

Answer 0.348 [3]

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(Questions continue overleaf)

[Turn over

5 Throughout the year Alice took nine written tests and five practical tests in Music.

The mean mark of her nine written tests was 69

The mean mark of her five practical tests was 83

(i) Calculate the mean mark of all 14 tests.

written → TOTAL = 9 x 69 = 621

Practical → TOTAL = 5 x 83 = 415

1036

Mean = $\frac{1036}{14}$ = 74

Answer 74 [2]

The standard deviation of her nine written tests was 15.4

The standard deviation of her five practical tests was 7.8

(ii) Calculate the standard deviation of all 14 tests.

Written s.d → $\sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2} = 15.4$

$\sqrt{\frac{\Sigma fx^2}{9} - (69)^2} = 15.4$

$\frac{\Sigma fx^2}{9} - 4761 = 237.16$

$\Sigma fx^2 = 44983.44$

Practical s.d → $\sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2} = 7.8$

$\frac{\Sigma fx^2}{5} - 83^2 = 60.84$

$\Sigma fx^2 = 34749.2$

TOTAL $\Sigma fx^2 = 34749.2 + 44983.44 = 79732.64$

$sd = \sqrt{\frac{79732.64}{14} - 74^2}$

$= 14.805\,01845$

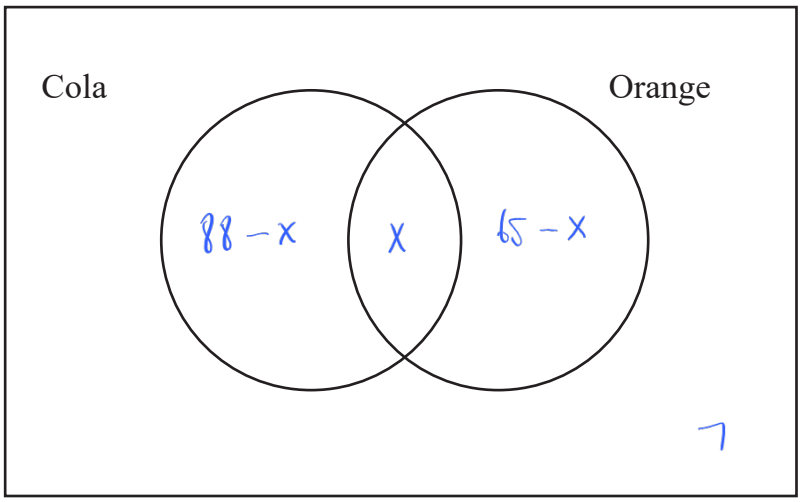
Answer 14.8 [4]

[Turn over

6 Students at a school were asked which drinks they drink regularly.

88% drink Cola
65% drink Orange
7% drink neither Cola nor Orange.

(i) Using the diagram below, or otherwise, find the percentage of students who drink both Cola and Orange.



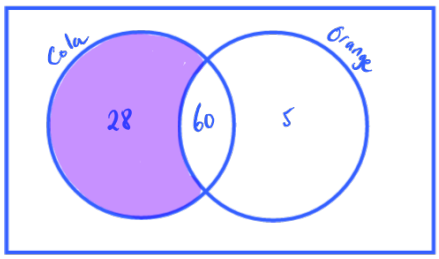
$88 - x + x + 65 - x + 7 = 100$

$160 - x = 100$

$x = 60$

Answer 60 % [2]

(ii) Find the probability that a student, selected at random, does not drink Orange, given that they drink Cola.



$P(\text{Not orange} \mid \text{Cola}) = \frac{28}{88} = \frac{7}{22}$

Answer _____ [2]

Two students are selected at random.

(iii) Find the probability that one of them drinks Cola only and the other one drinks Orange only, giving your answer to 3 decimal places.

$P(\text{Cola only then Orange only}) = \frac{28}{100} \times \frac{5}{100} = \frac{140}{10000}$

$P(\text{Orange only then Cola only}) = \frac{5}{100} \times \frac{28}{100} = \frac{140}{100\,000}$

$\text{TOTAL} = \frac{280}{100\,000} = \frac{7}{25000}$

Answer 0.028 [2]

[Turn over

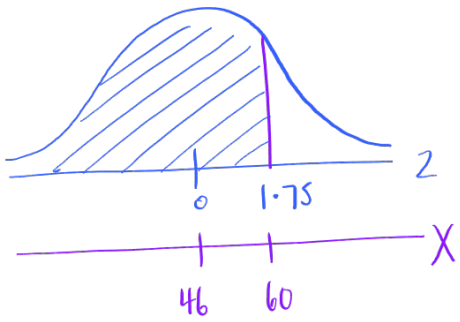
7 A teacher recorded the times taken to complete a cross-country race by Year 12 pupils.

The times were normally distributed with mean 46 minutes and standard deviation 8 minutes.

(i) Find the probability that a pupil, chosen at random, took less than 60 minutes to complete the race. Give your answer correct to 4 decimal places.

$$z = \frac{X - \mu}{\sigma} = \frac{60 - 46}{8} = 1.75$$

$$P(X < 60) = P(z < 1.75) = 0.9599$$

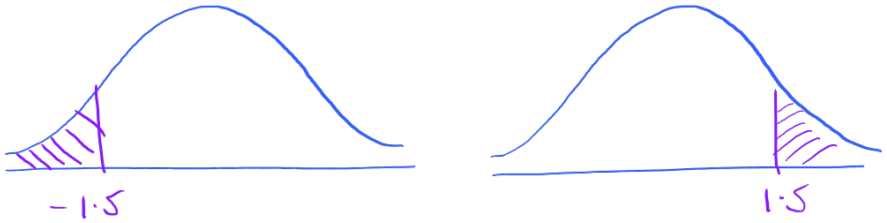


Answer 0.9599 [3]

(ii) Find the probability that a pupil, chosen at random, took less than 34 minutes to complete the race. Give your answer correct to 4 decimal places.

$$z = \frac{x-\mu}{\sigma} = \frac{34-46}{8} = -1.5$$

$$P(X < 34) = P(z < -1.5) = P(z > 1.5)$$



$$= 1 - P(z < 1.5)$$
$$= 1 - 0.9332$$

Answer 0.0668 [4]

(A)

(iii) Find the probability that a pupil, chosen at random, took less than 34 minutes to complete the race, given that they took less than 60 minutes to complete the race. Give your answer correct to 4 decimal places. (B)

$$P(A) = 0.0668$$
$$P(B) = 0.9599$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$= \frac{0.0668}{0.9599}$$

Answer 0.0696 [2]

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Question Number	**Marks**
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